International Journal of Thermal Sciences 110 (2016) 299–303

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Control of heat and mass transfer processes by plasma equilibrium reconstruction in toroidal magnetic confinement nuclear fusion devices

Z. Amerian [a], M.K. Salem [a, *], A. Salar Elahi [a], M. Ghoranneviss [a], J. Izadiyan [b]

[a] Plasma Physics Research Center, Science and Research Branch, Islamic Azad University, Tehran, Iran
[b] Department of Physics, Mashhad Branch, Islamic Azad University, Mashhad, Iran

A R T I C L E I N F O

Article history:
Received 12 June 2016
Received in revised form
26 July 2016
Accepted 27 July 2016
Available online 5 August 2016

Keywords:
Tokamak
Heat transfer
Grad-Shafranov equation

A B S T R A C T

Heat and mass transfers can be controlled by plasma equilibrium reconstruction in toroidal confinement nuclear fusion systems such as tokamaks. The Magnetohydrodynamic equilibrium in axisymmetric plasma is described by the Grad-Shafranov equation in terms of the magnetic flux. In this paper, we have proposed a new numerical solution to the Grad-Shafranov equation of an axisymmetric, transformed in quasi-cylindrical coordinates solved with the Chebyshev collocation method, when the source term (current density function) on the right hand side is quadratics as it is described by Atanasiu et al. The Chebyshev collocation method is a method for computing highly accurate numerical solutions differential equations. We have described a circular cross section of tokamak and presented numerical result of magnetic surfaces on the IR-T1 tokamak and compared the results with an analytical solution and then calculated the Shafranov shift using a minimization procedure based on the Newton method.
© 2016 Elsevier Masson SAS. All rights reserved.

1. Introduction

A plasma is an electrically conducting fluid or gas consisting totally or partially of charged particles. At high temperatures the highly ionized plasma is an excellent electrical conductor, and can be confined and shaped by strong magnetic fields. Particular plasma configurations are described in terms of solutions of the Grad-Shafranov equation. In a tokamak, external magnetic measurements have been applied to determine the important information on plasma shapes, the safety factor, the sum of the average poloidal beta β_P and internal inductance l_i [1–10]. There are methods for extraction of plasma parameters from external magnetic measurements. Swain and Neilson [2] presented an efficient method to reconstruct the plasma shapes and line integrals of the boundary poloidal magnetic field from external magnetic measurements. In their method, the plasma current distribution is approximated by using a few filament currents. In Luxon and Brown's approach [3], the plasma current is modeled using distributed sources. The non-linear Grad–Shafranov equation is solved repeatedly to search the best current density profile. If the plasma geometry possesses a symmetry property (axial or helical) a stream function can be introduced to describe the magnetic field. In the case of an axisymmetric torus, this leads to the well known Grad- Shafranov equation [11–20]:

$$\frac{\partial^2\psi}{\partial R^2}-\frac{1}{R}\frac{\partial\psi}{\partial R}+\frac{\partial^2\psi}{\partial Z^2}=-\mu_0 R^2\frac{dP}{d\psi}-\frac{1}{2}\frac{dF^2}{d\psi}, \tag{1}$$

where $2\pi\psi$ is the poloidal flux, $-2\pi F=I_P$ is the net poloidal current flowing in the plasma and the toroidal field coils, and $P=P(\psi)$ is the thermal pressure. Note that the toroidal field is not determined by the Grad– Shafranov equation. $F^2(\psi)$ and $P(\psi)$ are arbitrary functions of ψ which occur as source terms on the right hand side of the Grad-Shafranov equation. The simplest analytic solutions are obtained with pressure and current profiles which are linear in the flux function of ψ, the so-called Solov'ev profiles. These solutions have been extensively studied (e.g. Refs. [4,6]), and have given very useful insights, for instance in the study of plasma shaping effects in spherical tokamaks. Unfortunately, the Solovev's profiles correspond to the unrealistic situation where the toroidal current has a jump at the plasma edge [21–34]. We focus on pressure and current profiles which are quadratic. In this work, a method is proposed to compute highly accurate numerical solution of the Grad-Shafranov

* Corresponding author.
E-mail address: m.salem@srbiau.ac.ir (M.K. Salem).

http://dx.doi.org/10.1016/j.ijthermalsci.2016.07.014
1290-0729/© 2016 Elsevier Masson SAS. All rights reserved.

equation by using the Chebyshev collocation method. We have described a circular cross section of tokamak and present numerical result of magnetic surfaces on the IR-T1 tokamak and compare the results with an analytical solution and then calculate the Shafranov shift using a minimization procedure based on the Newton method.

2. Approximation of the magnetic surfaces based on analytical solution

The solutions of Grad–Shafranov equation analytically can be used for theoretical studies of plasma equilibrium, transport, and Magnetohydrodynamic stability. The existing exact solutions have arisen from a variety of allowed current density profiles or a variety functional of source functions. In this section we apply the analytic solution to the Grad-Shafranov. The several analytic solutions that exist depend on specific choices for $F^2(\psi)$ and $P(\psi)$ which occur in source term on the right hand side of the Grad-Shafranov equation. We have chosen the arbitrary functions $P(\psi)$ and $F^2(\psi)$ as quadratic function of ψ as also suggested by Atanasiu et al. [8] and Guazzotto et al. [9]. Specifically, we write:

$$P(\psi) = a^2 P_{axis}\left(\psi^2 \Big/ \psi^2_{axis}\right),$$

$$F^2(\psi) = a^2 R_0^2 B_0^2 \left(1 + b_{axis}\left(\frac{\psi^2}{\psi^2_{axis}}\right)\right). \tag{2}$$

where R_0 and a are major radius and minor radius of torus, respectively. Here P_{axis},ψ_{axis} and b_{axis} are constants related to the values P,ψ and F^2 of on axis andB_0 is the vacuum toroidal field at the geometric center of the plasma. Note that for a vacuum toroidal field $b_{axis} = 0$ and $B_\varphi = B_0(R_0/(R_0 + rcos\theta))$. With plasma pressure the toroidal field is given by $B_\varphi = B_0(R_0/(R_0 + rcos\theta))(1 + b_{axis}\psi^2/\psi^2_{axis})^{1/2}$.

On the magnetic axis $R = R_{axis}$, $\psi = \psi_{axis}$ giving $B_\varphi = B_{axis}(1 + b_{axis})^{1/2}$. Here $B_{axis} = B_0(R_0/R_{axis})$ is the vacuum field at $R = R_{axis}$. We see that b_{axis} is a measure of the plasma diamagnetism (if b_{axis}<0) or paramagnetism (if $b_{axis} > 0$).

The Grad-Shafranov equation reduces to

$$\frac{\partial^2\psi}{\partial R^2} - \frac{1}{R}\frac{\partial\psi}{\partial R} + \frac{\partial^2\psi}{\partial Z^2} = -\left(\frac{a^2 R_0^2 B_0^2}{\psi^2_{axis}}\right)\left(b_{axis} + \beta_{axis}\frac{R^2}{R_0^2}\right)\psi, \tag{3}$$

where $\beta_{axis} = 2\mu_0 P_{axis}/B_0^2$. The next step is to introduce normalized variables: $R^2 = R_0^2 x$ and $Z = ay$. Equation (3) can be rewritten as

$$4\in^2 x\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + (\alpha x + \gamma)\psi = 0, \tag{4}$$

where $\varepsilon = a/R_0$ and

$$\gamma = \left(\frac{aR_0B_0}{\psi_{axis}}\right)^2 b_{axis}\ ,$$

$$\alpha = \left(\frac{aR_0B_0}{\psi_{axis}}\right)^2 \beta_{axis}. \tag{5}$$

The parameters α andγ also have physical meanings, with α~β_P, where β_P is the poloidal beta, and γ~$2(q/\varepsilon)^2(\delta B_\phi/B_\phi)$ is the normalized diamagnetism. These connections to the physical quantities of interest make it possible to choose reasonable values for α and γ [9].

The solution to Equation (4) is found by separation of variables:

$$\psi = \sum_m X_m(\rho)Y_m(y), \tag{6}$$

with $x = -i(\in/\sqrt{\alpha})\rho$ and for up-down symmetric case

$$Y_m(y) = \cos(k_m y), \tag{7}$$

Here, k_m is the m^{th} separation constant. The $X_m(\rho)$ equation reduces to:

$$\frac{d^2X_m}{d\rho^2} + \left[-\frac{1}{4} + \frac{\lambda_m}{\rho}\right]X_m = 0, \tag{8}$$

The solutions for $X_m(\rho)$ of Equation (8) are Whittaker functions [10].

$$X_m(\rho) = a_m W_{\lambda_m,\mu}(\rho) + b_m M_{\lambda_m,\mu}(\rho). \tag{9}$$

The a_m and b_m are unknown expansion coefficients and in this model $\mu = 1/2$. Guazzotto proposed only three terms for m, and then ψ can be written as:

$$\psi(\rho, y) = \sum_{m=1}^{3}\left(a_m W_{\lambda_m,\mu}(\rho) + b_m M_{\lambda_m,\mu}(\rho)\right)\cos(k_m y). \tag{10}$$

where a_m, b_m and k_m are nine unknown coefficients which must be determined. By setting$k_1 = 0.13$, $k_2 = 0.12$ and $k_3 = 0.10$. The first six of them can be obtained from boundary conditions. For boundary condition we assume that $\psi = 1$ on the plasma surface. The IR-T1 tokamak is a small air-core transformer tokamak with circular cross section and without conducting shell and divertor where $R_0 = 45$ cm, $a = 12.5$ cm. Numerical results of magnetic surfaces on the circular cross section of tokamak as IR-T1 are displayed in Fig. 1.

3. Chebyshev collocation formulation of the Grad-Shafranov equation with quadratic pressure and current profiles

We consider a circular cross section of tokomak which can be presented in plane (R,Z) as the following domain.

$$\Omega = \left\{(R,Z)\in\mathbb{R}^2 | (R - R_0)^2 + Z^2 \le a^2\right\}, \tag{11}$$

In order to solve Equation (1), the following quasi-cylindrical coordinates transformation is considered, i.e.

$$\begin{cases} R = R_0 + rcos\theta \\ Z = r\sin\theta \end{cases} \quad 0 \le r \le a, 0 \le \theta \le 2\pi. \tag{12}$$

The cross section in quasi-cylindrical coordinates system is shown in Fig. 2.

By considering these coordinate system the Equation (1)

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\psi - \left(\frac{\cos\theta}{R_0 + rcos\theta} - \frac{1}{r}\right)\frac{\partial\psi}{\partial r} + \frac{\sin\theta}{r(R_0 + rcos\theta)}\frac{\partial\psi}{\partial\theta}$$
$$= -\mu_0(R_0 + rcos\theta)^2\frac{dP}{d\psi} - \frac{1}{2}\frac{dF^2}{d\psi}, \tag{13}$$

By substituting relations (2) into Equation (13) this equation can be written as

Download English Version:

https://daneshyari.com/en/article/668298

Download Persian Version:

https://daneshyari.com/article/668298

Daneshyari.com