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# Weighted average of inlet and outlet temperatures in borehole heat exchangers

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## HIGHLIGHTS

- An improved method analyzes thermal response tests in borehole heat exchangers.
- Proposed model improves estimate the local borehole thermal resistance.
- Method has been verified with field data sets on borehole heat exchangers.
- Model matches both short-time and long-time data in thermal response tests.
- Proposed method calculates average fluid temperature over the depth of a borehole.

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## ABSTRACT

Vertical borehole heat exchangers are used to couple heat pumps to the ground, which serves as a source or sink of heat. These ground source heat pump systems heat and cool buildings efficiently with low maintenance costs. Many heat transfer models use the mean of the inlet and outlet circulating fluid temperatures as an average temperature along the entire borehole length. In this paper a weighting factor for the inlet and outlet temperatures has been developed that can be combined with 1D radial models in order to account for the variations in temperature with depth. The proposed method gives more accurate results than the mean temperature approximation without requiring computationally intensive 3D models. The method has been verified with measured data from thermal response tests on boreholes with single and double U-tubes, as well pipe-in-pipe (coaxial) boreholes. On the other hand, the usual mean temperature approximation sometimes leads to significant errors and unphysical temperatures.



## 1. Introduction

Ground source heat pump (GSHP) systems are used to efficiently heat and cool buildings while incurring low maintenance costs [1]. Lund and Boyd [2] report an increase of over 300% in the number of installed units between 2005 and 2015. Approximately 4.19 million equivalent units are installed worldwide based on a 12 kW baseline (typical size for USA and Western European homes). Heat pumps exchange heat with the ground through a circulating fluid often in a closed loop that includes buried pipes in boreholes or trenches [1]. When a vertical borehole is used, the pipe configurations within the borehole may be a single U-tube, a set of double U-tubes or a pipe-in-pipe (coaxial) arrangement. In this paper we consider all three borehole arrangements.

The most common configuration for a vertical borehole heat exchanger (BHE) in the United States is single U-tube, which has a cross section as illustrated in Fig. 1a. Entering fluid flows down one pipe and flows up the other pipe. Often a grout mixture is placed in the borehole to fill the space between the outer pipe walls and the borehole wall. Many local governments in the United States require grout to prevent water and contaminants from migrating vertically along the borehole. In other locations exceptions may occur, such as in Sweden, where groundwater fills the space between the pipes and borehole wall [3].

Analysis methods of vertical BHEs often use the simple mean of inlet and outlet temperatures of the circulating fluid to represent an average temperature along the length of the borehole. That is,

$$T_m = \frac{T_{in} + T_{out}}{2} \quad (1)$$

Then, a transient 1D radial model estimates the heat transfer exchange between the circulating fluid and the ground. One such

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**Nomenclature**

| | |
|---|---|
| $a_1, a_2$ | coefficients |
| $c$ | volumetric heat capacity, J/(K m$^3$) |
| $c_p$ | mass heat capacity, J/(K kg) |
| $C_1, C_2, C_3$ | coefficients |
| $C_4, C_5, C_6$ | coefficients |
| $f$ | weighting fraction |
| $h$ | convective film coefficient, W/(K m$^2$) |
| $H$ | ratio of volumetric heat capacities |
| $J_n(u)$ | Bessel function of the first kind of order $n$ with argument $u$ |
| $k$ | thermal conductivity, W/(K m) |
| $K_n(u)$ | modified Bessel function of the second kind of order $n$ with argument $u$ |
| $L$ | length of borehole, m |
| $N$ | dimensionless thermal conductance |
| $Q$ | heat input rate, W |
| $r$ | radial coordinate or radius, m |
| $R$ | thermal resistance, (K m)/W |
| $Re$ | Reynolds number |
| $s$ | laplace transform variable |
| $t$ | time, s |
| $T$ | temperature, °C |
| $\overline{T}_D$ | laplace transform of dimensionless temperature |
| $V$ | volume, m$^3$ |
| $w$ | volumetric fluid flow rate, m$^3$/s |
| $Y_n(u)$ | Bessel function of the second kind of order $n$ with argument $u$ |
| $z$ | vertical depth coordinate, m |

*Greek*

| | |
|---|---|
| $\alpha$ | thermal diffusivity, m$^2$/s |
| $\gamma$ | coefficient |
| $\mu$ | viscosity, kg/(m s) |
| $\rho$ | density, kg/m$^3$ |

*Subscripts*

| | |
|---|---|
| *avg* | average |
| *b* | borehole |
| *D* | dimensionless |
| *eq* | equivalent |
| *f* | circulating fluid |
| *g* | grout |
| *in* | borehole entrance |
| *m* | mean temperature approximation |
| *out* | borehole exit |
| *old* | old value |
| *s* | ground (or soil) |
| *sf* | steady flux |
| *tr* | transient time |
| *wa* | weighted average |
| 1 | pipe number 1 |
| 2 | pipe number 2 |
| 10 | 10% threshold |

model treats the borehole as a line source of heat [4,5]. Another approach is taking the finite diameter of the borehole into account with a cylindrical-source model [6–8]. These simplified models implicitly assume the heat transfer rate is uniform along the length of the borehole. More sophisticated models have been developed to capture some of the details of the borehole pipe geometry and the thermal properties of the circulating water and grout, which differ from the ground properties. For instance, models by Shonder and Beck [9] and Yavuzturk and Spitler [10] are more comprehensive, but these models still treat the fluid temperature as uniform over the borehole length. Other approaches [11,12] have combined detailed 2D numerical or network models with a model of the loop to simulated short and medium timescales. Reviews of models for vertical boreholes have been written by Yang et al. [13] and Li and Lai [14].

Acuña [3], Acuña and Palm [15] and Acuña et al. [16] have measured vertical temperature profiles of the circulating fluid in both U-tube and coaxial installations. They carry out the temperature measurements with a fiber optic cable placed inside both flow paths using an experimental method called Distributed Thermal Response Test (DTRT). Measured profiles are substantially different from the temperature profiles corresponding to the mean temperature approximation.

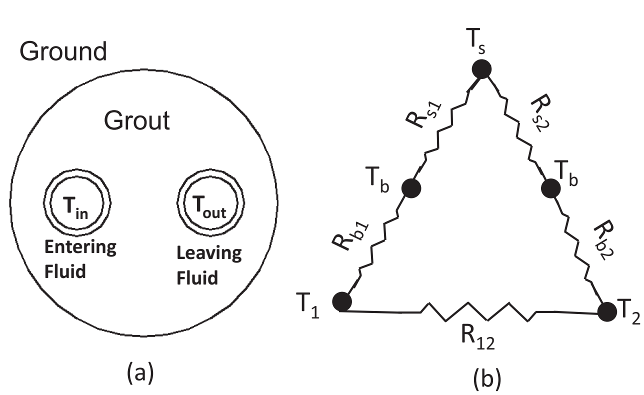


**Fig. 1.** (a) Borehole cross section and (b) thermal resistive network for single U-tube BHE.

A number of investigators [17–21] have demonstrated the mean of the inlet and outlet temperatures (Eq. (1)) may produce significant errors as the borehole length increases or the circulating flow rate decreases. The error occurs when this mean temperature is used with the local borehole thermal resistance, which may be estimated from the thermal properties of the circulating fluid, pipe materials and grout. The local borehole resistance $R_b$ has components shown in Fig. 1b for a single U-tube BHE. The resistance network in Fig. 1b applies at any given depth along the borehole. Heat transfer occurs between the circulating fluid and the surrounding ground through two resistances in series. The fluid enters pipe 1 where the resistance $R_{b1}$ represents a local borehole thermal resistance between the circulating fluid and the borehole wall. This resistance has contributions from the convective film in the pipe, the pipe wall and the grout. The second resistance $R_{s1}$ is the ground (or soil) resistance associated with heat conduction from the borehole wall through the ground. Similar resistances are illustrated for the second pipe. For symmetrically placed tubes within the borehole, $R_{b1} = R_{b2} = 2R_b$, where $R_b$ is the usual local borehole resistance representing the resistance between the circulating fluid in both pipes and the borehole wall. Heat transfer between pipes takes place through the local resistance $R_{12}$.

Deviations from the mean temperature approximation have been accounted for with two different simplified approaches. Hellström [17] defined the effective borehole thermal resistance, $R_b^*$, which is based on this simple mean temperature. Hellström [17] derived analytical expressions that relate the effective borehole thermal resistance to the local borehole thermal resistance for two special cases – borehole wall at a uniform temperature and a uniform heat flux over the borehole. The effective borehole