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Effects of variable thermal conductivity and fractional order of heat transfer on a perfect conducting infinitely long hollow cylinder

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ABSTRACT

A fractional model of the equations of generalized magneto-thermoelasticity for a perfect conducting isotropic thermoelastic media which is assumed to have variable thermal conductivity depending on the temperature is given. This model is applied to solve a problem of an infinite long hollow cylinder in the presence of an axial uniform magnetic field. The solution is obtained by a direct approach. Laplace transform techniques are used to derive the solution in the Laplace transform domain. The inversion process is carried out using a numerical method based on Fourier series expansions. Numerical computations for the temperature, the displacement and the stress distributions as well as the induced magnetic and electric fields are carried out and represented graphically. The results indicate that the thermal conductivity and time-fractional order play a major role in all considered distributions.
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1. Introduction

Heat transfer continues to be a field of major interest to engineering and scientific researcher, as well as designers, developers, and manufacturers. Considerable efforts have been devoted to research in traditional applications such as chemical processing, general manufacturing, and energy devices, including general power systems, heat exchangers, and high performance gas turbines [1].

Extensive research carried out in last few decades has led to the understanding that the classical theory of heat conduction, based on the Fourier law, predicts that a thermal disturbance at some point in a material body will be felt instantly at all the points of the body, however distant. This implies that thermal signals propagate with have been proposed in order to eliminate this "so-called paradox" inherent in the classical heat conduction theory. Several efforts have also been made to remove the "so-called paradox" inherent in the classical coupled dynamical theory of thermoelasticity introduced by Biot [2] that is based on the Fourier law of heat conduction. We recall the pioneering contributions by Lord and Shulman [3] as alternative theory of thermoelasticity that account for the finite speed of the thermal signal. In the first model, the heat conduction law was replaced with the Cattaneo–Vernotte heat conduction model [4,5] that includes one thermal relaxation time parameter.

A systematic development in the heat conduction theory and also the thermoelasticity theory can be found out in the review articles/books [6–11] which have reported several studies concerning applicability of these non-classical thermoelasticity theories. Within the theoretical contributions to the subject are the proofs of uniqueness theorems by Ezzat and El Karamany [12–15] and the boundary element formulation was done by El Karamany and Ezzat [16,17]. The propagation of discontinuities of solutions in these theories was investigated in Ref. [18]. A general solution to the field equations of generalized thermodiffusion in an elastic solid has been obtained by Ram et al. [19] and Ezzat et al. [20] has been established the model of the equations of generalized-thermoelasticity with thermal relaxation in an isotropic elastic medium with temperature-dependent mechanical properties.

Increasing attention is being devoted to the interaction between magnetic fields and strain in a thermoelastic solid due to its many applications in the fields of geophysics, plasma physics, and related topics. In the nuclear field, the extremely high temperatures and temperature gradients as well as the magnetic fields originating

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inside nuclear reactors influence their design and operation [21]. This is the domain of the theory of magneto-thermoelasticity. It is the combination of two different disciplines: those of the theories of electromagnetism and thermoelasticity. Among the authors who considered the generalized magneto-thermoelasticity equations are Nayfeh and Nasser [22] who studied the propagation of plane waves in a solid under the influence of an electromagnetic field. Choudhuri [23] extend these results to rotating media. El Karamany and Ezzat [24,25] proved the uniqueness and reciprocal theorems in linear micropolar electro-magnetic thermoelasticity. Sherief and Ezzat [26] solved a problem for an infinitely long annular cylinder, while Ezzat [27] obtained the fundamental solution in generalized magneto-thermoelasticity for perfect conductor cylindrical region. A two-dimensional generalized electro-magneto-thermoviscoelastic problem for a half-space with diffusion has been studied by Deswal and Kalkal [28]. Ezzat and El Karamany [29,30] solved some problems for an electric conducting media by using state space approach which proposed by Ezzat [31,32].

Fractional calculus has been used successfully to modify many existing models of physical processes. One can state that the whole theory of fractional derivatives and integrals was established in the 2nd half of the 19th century. Caputo and Mainardi [33] found good agreement with experimental results when using fractional derivatives for description of viscoelastic materials and established the connection between fractional derivatives and the theory of linear viscoelasticity. Adolfsson et al. [34] constructed a new fractional order model of viscoelasticity. Povstenko [35] made a review of thermoelasticity that uses fractional heat conduction equation and proposed and investigated new models that use fractional derivative.

Recently, the fractional order theory of thermoelasticity was derived by Ezzat [36–39]. It is a generalization of both the coupled and the generalized theories of thermoelasticity. El Karamany and Ezzat [40,41] introduced two general models of fractional heat conduction law for a non-homogeneous anisotropic elastic solid. Uniqueness and reciprocal theorems are proved and the convolutional variational principle is established and used to prove a uniqueness theorem with no restriction on the elasticity or thermal conductivity tensors except symmetry conditions. One can refer to Ezzat et al. [42–48] and Rastovic [49,50] for a survey of applications of fractional calculus.

The aim of this article is to study the effects of the derivative fractional order and the variable thermal conductivity in heat transfer on the temperature, displacement, and stresses for a problem of an infinite perfect conducting long hollow cylinder in the presence of an axial uniform magnetic field by using the fractional order theory of thermoelasticity. Laplace transform techniques are used. The inversion of the Laplace transforms is carried out using a numerical approach based on Fourier series expansions [51]. Numerical results for the temperature, stress, and displacement distributions are given and illustrated graphically for the given problem.

2. Mathematical model

We shall consider a thermoelastic medium of perfect conductivity permeated by an initial magnetic field $\boldsymbol{H_o}$. Due to the effect of this magnetic field there arises in the conducting medium an induced magnetic field $\boldsymbol{h}$ and induced electric field $\boldsymbol{E}$. Also, there arises a force $\boldsymbol{F}$ (the Lorentz Force). Due to the effect of the force, points of the medium undergo a displacement vector $\boldsymbol{u}$, which gives rise to a temperature. The linearized equations of electromagnetism for slowly moving media [27]

$$\text{Curl } \boldsymbol{h} = \boldsymbol{J} + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \tag{1}$$

$$\text{Curl } \boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{h}}{\partial t} \tag{2}$$

$$\boldsymbol{E} = -\mu_0 \frac{\partial \boldsymbol{u}}{\partial t} \wedge \boldsymbol{H}_0 \tag{3}$$

$$\text{div } \boldsymbol{h} = 0 \tag{4}$$

where t is the time, and μ_0, ε_0 are magnetic and electric permeability, respectively.

The above equations are supplemented by the displacement equations of the theory of generalized thermoelasticity, taking into account the external body force, which is here equal to the Lorentz force

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (\lambda + \mu) u_{j,ij} + \mu u_{i,jj} - \gamma T_{\ ,i} + \mu_0 (\boldsymbol{J} \wedge \boldsymbol{H}_0)_i \tag{5}$$

where ρ is the density, λ,μ are Lamé's constants, T is the absolute temperature, J is the electric current density vector, α is the fractional parameter, $\gamma = (3\lambda+2\mu)\,\alpha_T$ and α_T is the coefficient of linear thermal expansion.

The generalized fractional heat conduction equation [52]

$$\left(kT_{,i}\right)_{,i} = \frac{\partial}{\partial t}\left(1 + \frac{\tau^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\left(\rho C_E T + \gamma T_0 u_{i,i}\right) \tag{6}$$

where k is the thermal conductivity, C_E is the specific heat at constant strain, τ is relaxation time, T_0 is the temperature of the medium in its natural state, assumed to be such that $\frac{|T-T_0|}{T_0} < < 1$.

The constitutive equation

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - \gamma (T - T_0) \tag{7}$$

whereσ_{ij} are the components of stress tensor, e_{ij} are the components of strain tensorδ_{ij} is the Kronecker delta.

The strain-displacement relations

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{8}$$

together with the previous equations, constitute a complete system of fraction generalized magneto-thermoelasticity equations for a medium with perfect electrically conductivity.

3. The physical problem

Let (r, ψ, z) be cylindrical coordinates with the z-axis coinciding with the axis of a solid infinitely long hollow elastic circular cylinder of a homogeneous, isotropic material with a perfect electric conductivity of internal radius (a) and external radius (b). The outer surface of this cylinder is assumed to be traction free and subject to a thermal shock that depends only on the time t, while the inner surface is assumed to be in contact with a rigid surface and is thermally insulated. Assume also that the initial magnetic field $\boldsymbol{H_o}$ acts in the direction of the z-axis and has the components $(0, 0, H_o)$. The induced magnetic field $\boldsymbol{h}$ will have one component h in the z-direction, while the induced electric field $\boldsymbol{E}$ will have one component E in the ψ-direction. Because of the cylindrical symmetry of the problem, and if there is no z-dependence of the field variables, all the considered functions will be functions of r and t only (Scheme 1).

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