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A new heat transfer correlation for transition and turbulent fluid flow in tubes

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ABSTRACT

The objective of the paper is to develop a correlation for the Nusselt number Nu in terms of the friction factor ξ (Re), Reynolds number Re, and also Prandtl number Pr, which is valid for transitional and fully developed turbulent flow. After solving the equations of energy conservation for turbulent flow in a circular tube subject to a uniform heat flux, the Nusselt number values were calculated for different values of Reynolds and Prandtl numbers. Then, the form of the correlation $Nu = f(Re, Pr)$ was selected which approximates the results obtained in the following ranges of Reynolds and Prandtl numbers: $2300 \le Re \le 10^6$, $0.1 \le Pr \le 1000$. The form of the correlation was selected in such a way that for the Reynolds number equal to $Re = 2300$, i.e. at the point of transition from laminar to transitional flow the Nusselt number should change continuously. Unknown coefficients $x_1,\ldots,x_n$ appearing in the heat transfer correlation expressing Nusselt number as a function of the Reynolds number and Prandtl number were determined by the method of least squares. To determine the values of the coefficients at which the sum of the difference squares is a minimum, the Levenberg–Marquardt method is used. The proposed correlation was validated by comparing with experimental data.

1. Introduction

Heat transfer correlations based on the experimental data are widely used to calculate mean heat transfer coefficients in heat exchangers and other thermal installations. A common way to find these correlations involves performing heat transfer measurements and correlating the data in terms of appropriate dimensionless numbers, which are obtained by expressing mass, momentum, and energy conservation equations in dimensional forms or from the dimensional analysis. A functional form of the relation $Nu = f(Re,Pr)$ is usually based on energy and momentum-transfer analogies. Traditional expressions for calculation of heat transfer coefficient in fully developed flow in smooth tubes are usually products of two power functions of the Reynolds and Prandtl numbers.

Until recently, the Dittus-Boelter correlation for turbulent flow in tubes has been widely used [1–3]. The Dittus-Boelter relationship, as introduced by McAdams [2,3], is

$$Nu = 0.023Re^{0.8}Pr^{n}, \quad 0.7 \le Pr \le 120, \quad Re \ge 10^4, \quad L/d_w \ge 60 \tag{1}$$

The exponent of the Prandtl number is $n = 0.4$ for heating of the fluid and $n = 0.3$ if the fluid is being cooled. The Dittus-Boelter correlation has a real physical basis since a similar equation can be obtained using the Chilton-Colburn analogy [4,5],

$$j = \frac{\xi}{8} \tag{2}$$

where the Colburn factor j is defined as

$$j = \frac{Nu}{RePr^{1/3}} \tag{3}$$

Substituting the Moody equation for the friction factor in smooth tubes [6],

$$\xi = \frac{0.184}{Re^{0.2}}, \quad Re \ge 10^4 \tag{4}$$

into Eq. (2) gives the relation proposed by Colburn

$$Nu = 0.023Re^{0.8}Pr^{1/3}, \quad 0.7 \le Pr \le 160, \quad Re \ge 10^4, \quad L/d_w \ge 60 \tag{5}$$

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Nomenclature

c_p	specific heat at constant pressure, J/(kg K)
c_1,c_2	constants
d_h	hydraulic diameter of a duct, m
d_w	inner diameter of a circular tube, $d_w = 2r_w$, m
e_i	relative difference
FD	fully developed flow
i	node number
j	Colburn parameter, $j = Nu/(RePr^{1/3})$
k	thermal conductivity, W/(m K)
k	turbulence kinetic energy, N/(s m^2)
L	tube length, m
n	number of nodes in the finite difference grid
Nu	Nusselt number, $Nu = hd_w/k$
$Nu_{m,q}$	mean Nusselt number over the tube length in laminar flow for constant wall wall heat flux
$Nu_{m,T}$	mean Nusselt number over the tube length in laminar flow for constant wall temperature
p	pressure, Pa
Pr	Prandtl number, $Pr = c_p\mu/k$
PKN	Prandtl-von Kármán-Nikuradse
q	heat flux, W/m^2
q_m	molecular heat flux, W/m^2
q_t	turbulent heat flux, W/m^2
q_w	heat flux at the inner surface of the tube, W/m^2
r	radial coordinate, m
r_w	inner radius of the tube, m
r^2	coefficient of determination
r^+	dimensionless radius, $r^+ = ru_\tau/\nu$
R	dimensionless radius, $R = r/r_w$
Re	Reynolds number, $Re = w_m d_w/\nu$
SE	square-edged (sudden contraction) inlet to the tube
T	temperature, °C or K
T'	fluctuating component of the local temperature in turbulent flow, K
$\overline{T}_1(x)$	time and mass averaged fluid temperature, °C or K
$\overline{T}_2(r)$	time averaged radial component of the temperature, °C or K
u_τ	friction velocity, $u_\tau = \sqrt{\tau_w/\rho}$, m/s
UHF	uniform heat flux
UWT	uniform wall temperature
w_m	mean velocity
w'_r	fluctuating component of the radial velocity in turbulent flow, m/s
w_x	velocity component in the x direction, m/s
$\overline{w}_x$	time averaged velocity component in the x direction, m/s
x	a spatial coordinate in Cartesian or cylindrical coordinate systems or distance from the tube inlet, m
y	a spatial coordinate in a Cartesian system or distance from distance from the wall surface m
y^+	dimensionless distance from the tube wall, $y^+ = yu_\tau/\nu$

Greek symbols

Δy^+	dimensionless spatial step
Γ	Gamma function
ε	turbulence dissipation rate, N/(s m^2)
ε_q	eddy diffusivity for heat transfer, m^2/s
ε_τ	eddy diffusivity for momentum transfer (turbulent kinematic viscosity), m^2/s
μ	dynamic viscosity, kg/(m s)
ν	kinematic viscosity, $\nu = \mu/\rho$, m^2/s
ξ	Darcy-Weisbach friction factor
ρ	fluid density, kg/m^3
τ	shear stress, Pa
τ_w	shear stress at wall surface, Pa

Subscripts

b	bulk
m	mean
i	node number
w	wall surface

Superscripts

$-$	time averaged
$+$	dimensionless

The empirical correlations of Dittus-Boelter and Colburn have gained widespread acceptance for prediction of the Nusselt number with turbulent flow in the smooth-surface tubes. However, the Dittus-Boelter and Colburn equations do not provide a good correlation of the experimental data because of their simple form. The power type correlations like those of Dittus-Boelter and Colburn are not able to approximate the experimental data over a broad range of the Prandtl number. The maximum deviation between experimental data and predictions using Eq. (1) or Eq. (5) is about 20% [7–9]. The smooth-tube heat transfer results reported by Allen and Eckert [8] were obtained for developed turbulent flow of water under the uniform wall heat flux boundary condition at $Pr = 7$ and $Pr = 8$, and $1.3 \cdot 10^4 \le Re \le 1.11 \cdot 10^5$ [8]. Nusselt numbers determined experimentally were 10–20% higher than values predicted by the Dittus-Boelter equation. The discrepancy between the results of measurements and values of Nusselt number obtained from the Dittus-Boelter formula is greater for higher Reynolds numbers. In another experimental study, the Dittus-Boelter relationship (1) underpredicted the data by 5–15% for turbulent water flow through a tube at Prandtl numbers of 6.0 and 11.6 [9]. The Reynolds number ranged between 10^4 and 10^5. On the other hand, power-type correlations are still used successfully to approximate the experimental data [10–14], when the Reynolds and Prandtl numbers vary in a narrow range. Siddique et al. [10] studied turbulent flow and heat transfer inside a micro-finned tube. The heat transfer data for $3.3 \cdot 10^3 \le Re \le 2.25 \cdot 10^4$ and $2.9 \le Pr \le 4.7$ were correlated using a relationship of the Dittus-Boelter type. Zhang et al. [11] carried out the thermo-hydraulic evaluation of the heat transfer enhancement in the smooth tubes fitted with rotor-assembled strands of various diameters. A power type correlation for the Nusselt number was developed with the Reynolds number ranging from $2.5 \cdot 10^4$–$7.5 \cdot 10^4$ and the Prandtl number ranging from 4.188 to 4.274. Experimental [12,14] and numerical [13] studies were conducted to investigate enhancement in heat transfer by using different nanofluids. Results of an experimental study on convective heat transfer of non-Newtonian nanofluids flowing through a uniformly heated tube under turbulent flow conditions are presented by Hojjat et al. [12]. They proposed a new power-type correlation for prediction of the Nusselt number of nanofluids. The Reynolds number varied from 2800–8400 and the Prandtl number from 40–73. Moghadassi et al. [13] conducted a CFD simulation of a laminar flow in a horizontal tube to investigate the effect of