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A genetic algorithm for topology optimization of area-to-point heat conduction problem

R. Boichot [a, b, *], Y. Fan [c]

[a] Univ. Grenoble Alpes, SIMAP, F-38000 Grenoble, France
[b] CNRS, SIMAP, F-38000 Grenoble, France
[c] LTN Nantes, La Chantrerie, rue Christian Pauc BP 90604, 44306 Nantes Cedex 3, France

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ABSTRACT

This paper presents a way of solving the classical area (volume)-to-point heat conduction problem by the means of a simple Genetic Algorithm (GA) in square configuration. After a short description of the numerical method, the optimal solutions proposed for minimizing the peak or mean temperature of a domain are presented. The effects of the conductivity ratio and the filling ratio on the configurations of the conductive tree are also analyzed and discussed. A numerical benchmark is then established to assess the influence of mesh resolution and the reproducibility of the GA optimization. Results show that GA is capable of proposing solutions having almost the same cooling effectiveness for different mesh resolutions or random seed generators. GA is also relevant compared to other optimization techniques presented here. It can be considered as a simple, easy to adapt and robust but computation time consuming method for addressing the general area (volume)-to-point heat conduction problem.

1. Introduction

The problem of cooling a continuous heat generating area (or volume) is widely recognized in electronic industries because the accumulation of generated heat, if not timely removed, will cause severely damage of the electronic components. One usual solution is to integrate a certain quantity of highly conductive material in the area (volume) to drain the generated heat to a heat sink (point) by pure heat conduction. The cooling effectiveness depends not only on the quantity (filling ratio, ϕ) and quality (conductivity ratio, k_p/k_0) of the available highly conductive material, but also on the topological configuration of the material to form the conducting path. How to determine the optimal configuration of the highly conductive material subjected to different objectives (e.g. minimum thermal resistance, minimum peak temperature, etc.) within the framework of the general "area (volume)-to-point" heat conduction problem has received a great attention in the literature.

The issue was analytically studied by proposing a "constructal approach" through first determining the shape of the optimal rectangular elemental areas and then assembling them scale by scale to pave the whole surface [1]. The final optimal shape of the conducting path turns out to be tree-like networks, not inferred by assumptions but deduced by optimization. The fundamental constructal theory [2,3] was followed and extended in various aspects through formulating different objective functions, releasing the constraints or adding degrees of freedom [4]. These extensive analytical efforts offer an elegant way of solving the basic area (volume)-to-point problem with reduced global thermal resistance and enhanced cooling effect. Nevertheless, the analytical method becomes mathematically difficult, sometimes impossible, when dealing with irregular or unspecified geometries.

So emerges the idea that classic analytical approaches may benefit from modern numerical computing by adding more degrees of morphologic freedom. The numerical methods consist in freely paving the surface to be topologically optimized with the only constraint of the meshing, and so, the computational power available. The first attempt in this direction was made by analogy with fluid flow in river drainage basins [5]. After, the bionic optimization method was developed based on a gradient attraction to enhance heat drain topology efficiency [6,7]. Within the same class of local attraction algorithm different authors [8,9,10], successively proposed cellular automaton (CA) algorithms driven by thermal gradients, by temperature or by both. It was reported that the CA algorithms (including bionic optimization) can offer a simple way

* Corresponding author. SIMAP, Phelma Bâtiment Recherche, 1130 rue de la piscine, 38402 Saint Martin d'Hères, France.
E-mail address: Raphael.boichot@simap.grenoble-inp.fr (R. Boichot).

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Nomenclature

A, R	Non-dimensional thermal resistances –
H	Height of the domain m
k_p	Thermal conductivity of highly conductive material W m^{-1}K^{-1}
k_0	Thermal conductivity of heat generating material W m^{-1}K^{-1}
k_{ad}	Thermal conductivity of adiabatic elements W m^{-1}K^{-1}
L	Size of police
T_{max}	Temperature of the hottest element of the domain K
T_{mean}	Mean temperature of the domain K
T_{sink}	Temperature of the heat sink K
p	Heat generation rate per unit volume or surface W m^{-3}
ϕ	Volume or surface fraction of high conductivity material

(in terms of computational efforts) to obtain a sub-optimal but acceptable solution to the area-to-point problem with rational values of conductivity ratio (k_p/k_0) [11]. Nevertheless, the drawbacks of the CA algorithms are apparent: no clue implies that the final configuration is the optimum, i.e. no objective function to minimize is explicitly used but only local attraction based on intuitive hypotheses.

More recently, a variety of numerical algorithms were proposed to tackle this problem. To list some, simulated annealing (SA) method [12], Solid Isotropic Material with Penalization (SIMP) [13], method of moving asymptotes (MMA) [14–15] and SIMP model with an aggregated objective function approach (AOF) [16] were used to tackle this topology optimization problem. A systematic and quantitative robustness analysis of solutions of various optimization algorithms was provided [17].

Meanwhile, another class of numerical optimization algorithms that is extensively used in heat transfer problems [18] -the Genetic algorithm (GA)-has rarely been applied for solving this heat conduction problem. Genetic algorithms, based on the Darwin's theory of evolution [19], are characterized by a poor sensitivity to local minima so that the global best solutions can theoretically be reached. Moreover, GA is well-adapted to objective functions with a huge number of parameters (or dimensions) and non-differentiable (discrete) problems. The pioneer work reported in literature [12] showed that GA proposed better solutions compared to those of bionic optimization, especially for high conductivity ratio conditions. Due to the computational time consuming nature of GA however, only a few cases were studied, with limited meshing fineness, limited values of conductivity ratio ($k_p/k_0 = 3$; 10; 100) and filling ratio ($\phi = 0.1$). In fact, besides the fitness (the objective function), a number of more intrinsic parameters in the sense of numerical methods (e.g. the mutation probability, the crossing probability, the rate of selection of the best individuals, etc.) have to be assessed to optimize the convergence of the GA. In-depth investigations and systematic analyses are still in need in this area due to the strong influence of these parameters on convergence.

The goal of this study is to formulate a GA for efficiently solving the area-to-point heat conduction problem on one hand, and to analyze the effects of various parameters of practical use on the evolved topology on the other hand. We shall first present the GA method with a general 2D case, appropriate for introducing the notations and for describing in detail the basic principles of optimization. Then, the performances of the algorithm will be assessed for the minimization of two different objective functions: the peak temperature and the mean temperature across the domain. The effects of different conductivity ratios ($k_p/k_0 = 2$; 10; 50; 250) and filling ratio ($\phi = 0.1$; 0.3; 0.5) on the optimal configuration of conductive paths will be analyzed and discussed. After that, a dedicated numerical benchmark will be developed to investigate the meshing sensitivity and the reproducibility of GA method. Comparison with other methods addressing this problem is also proposed. Finally, technical remarks and main conclusions will be summarized.

2. Genetic algorithm

The GA is coded using software suite Matlab, assisted by the Parallel Computing toolbox (www.mathworks.com/products/parallel-computing/). The basic idea is to imitate the natural selection and survival of the fittest that exists in the genetics of the species. A synthesis of GA principles, applications and examples can be find in literature [20].

2.1. Geometry and boundary conditions

The sketch of a typical area-to-point heat conduction problem to be solved by GA is given in Fig. 1. The entire domain was discretized into small and homogeneous square elements, each element having determinate conductivity, uniform temperature and heating rate. In this study, a mesh of 100 × 50 square elements was used, considering a compromise between calculation time and precision. Other mesh resolutions (12 × 25, 25 × 50 and 100 × 200) were also assessed and the effects of mesh resolution on the complexity of optimal conductive trees will be discussed in the later section.

Five kind of elements are defined to cover the entire calculation domain, as described below.

- Heat sink elements (blue): cells with a constant temperature ($T_{sink} = 293$ K) and a thermal conductivity equal to k_p. The isothermal heat sink has an aperture width of 20% of one side of the domain.
- Symmetry elements (red): the right boundary of the domain shown in Fig. 1 is defined as symmetry to represent a square

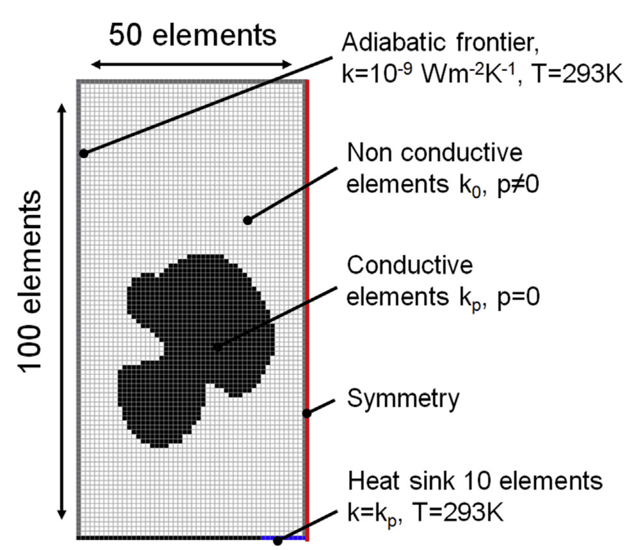

Fig. 1. Geometry and mesh used for this study. Note the symmetry on the right border.