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Convective heat transport by longitudinal rolls in dilute nanoliquid layer of finite depth

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ABSTRACT

The paper investigates the thermal stability of a horizontal layer of a Newtonian nanofluid in the linear and non-linear regimes. A finite depth of fluid layer is considered which incorporates the effect of Brownian motion and thermophoresis slip mechanism along with no nanoparticle flux boundary conditions. For the linear stability a normal mode analysis is performed whereas a two term Fourier series analysis has been used for the nonlinear analysis. In case of linear stability, analytic expression for thermal Rayleigh number (Ra) in terms of various pertinent parameters are obtained. The graphs are presented to visualize the effect of these parameters on the critical value of the thermal Rayleigh number (Ra_{cr}). Thermal Nusselt number in nonlinear analysis has also been plotted and discussed.

1. Introduction

In the late 19th century Maxwell [9], in his pioneering theoretical studies, proposed that the heat transfer properties of ordinary fluids can be enhanced by suspending metal particles in them. His claims were based on the fact that in comparison to any ordinary cooling fluid, the thermal conductivity of any metal is comparatively high. Though his idea had practical applications, it had many implications too. The large size of the particles could lead to clogging, drastic pressure drops, settling and premature wear on channels and components. These complications were reduced considerably by the use of nano-scale (1–100 nm) sized particles due to advent of technology of manufacturing them either physically or chemically. These render much larger relative surface area than micro-sized particles improving the heat transfer properties of ordinary coolants used till date. Choi [5,6] claimed the nanofluids as the next generation heat exchangers owing to the superior properties of these leading to savings of energy and cost, over the base fluids. Contrary to the milli- and microsized particle slurries explored in the past, nanoparticles are relatively close in size to the molecules of the base fluid, and thus can realize very stable suspensions with little gravitational settling over long periods of time.

With their very alluring properties, nanoliquids have fascinated physical as well as analytical researches in the last two decades or so. The nanoparticles used were copper, silver, gold, copper-oxide, alumina, SiC, in base fluids such as water, ethylene-glycol, toulene, etc.. The nanoparticle concentration used ranged from 0.11 vol % to 4.3 vol%, with the enhancement in thermal conductivity from 10% to 40%. Analytical researches claimed quite a few reasons for the observed phenomenon of thermal conductivity enhancement, but the two-component non-homogenous model proposed by Buongiorno [3] was accepted with less conflicts. Moreover, Magyari [8] also commented on homogenous nanofluid models (which neglected slip effects) applied to convective heat transfer. The non-homogenous Buongiorno's model was supported and extended by Tzou [15,16], Nield and Kuznetsov [10,11], Yadav et al. [17] and Agarwal and Bhadauria [1]. Tzou [15,16] studied the onset of convection in a horizontal layer of a nanofluid uniformly heated from below. Nield and Kuznetsov [10], Yadav et al. [17] and Agarwal and Bhadauria [1] investigated the fluid layer for convective stability under thermal equilibrium, whereas Nield and Kuznetsov [11] for local thermal non-equilibrium conditions.

Convective flows found widely in astrophysical, geophysical or industrial interest involve fluid layer. Thus studies on thermal convection of fluid layer have been undertaken by Chandrashekhar [4], Kim and Choi [7] and Shivakumara et al. [13,14]. Since we look upon nanofluids to be used in the above fields in future, study of nanofluids layer turns significant. So far, majority of the studies in the case of nanofluids were performed under the assumption that

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Nomenclature

Latin symbols

d	dimensional layer depth
p	pressure
Da	Darcy number
Pr	Prandtl number
N_A	modified diffusivity ratio
D_T	Thermophoretic diffusion coefficient
N_B	modified particle-density increment
D_B	Brownian Diffusion coefficient
Le	Lewis number
Rm	basic density Rayleigh number
Rn	concentration Rayleigh number
g	Gravitational acceleration
Ra	thermal Rayleigh number
T	nanofluid temperature
T_c	temperature at the upper wall
T_h	temperature t the lower wall
$\mathbf{v}$	nanofluid velocity
(x,y,z)	Cartesian coordinates
t	time

Subscripts

b	basic solution
f	Fluid phase.
p	Particle phase

Superscripts

$*$	dimensional variable
$'$	perturbation variable

Operators

∇^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$
∇_1^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}$

Greek symbols

β	proportionality factor
φ	nanoparticle volume fraction
μ	viscosity of the fluid
ω	frequency of oscillations
α	wave number
ψ	stream function
$(\rho c)_f$	Heat capacity of the fluid
$(\rho c)_p$	Heat capacity of the nanoparticle material
ρ_f	fluid density
ρ_p	nanoparticle mass density

the nanoparticle concentration can be controlled actively at the boundaries which is quite difficult to establish physically. Thus arose the need to modify the boundary conditions assumed. In their very recent work, Nield and Kuznetsov [12] came out with a new set of boundary condition which assume that there is no nanoparticle flux at the plate and that the particle fraction value there adjusts accordingly. With such an assumption, the presence of oscillatory convection turns oblivious due to the absence of the two opposing agencies affecting instability. The study was further promoted by Agarwal et al. [2].

In the present article, we have done both the linear and nonlinear stability analysis in a nanofluid layer under Rayleigh Be′nard problem, assuming that the nanoparticles being suspended in the nanofluid with no nanoparticle flux at the boundaries.

2. Governing equations

We consider a horizontal layer of nanofluid, confined between two horizontal boundaries at z = 0 and z = d, heated from below and cooled from above. The boundaries are impermeable and perfectly thermally conducting. The fluid layer is extended infinitely in x and y-directions, and z-axis is taken vertically upward with the origin at the lower boundary. T_h and T_c are the temperatures at the lower and upper walls respectively, where $T_h > T_c$. The resulting conservation equations will be [3]; [12]:

$$\nabla\cdot\mathbf{v}=0 \tag{1}$$

$$\rho_f\left(\frac{\partial\mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+\mu\nabla^2\mathbf{v}+\left[\varphi\rho_p+(1-\varphi)\left\{\rho_f\left(1-\beta\left(T_f-T_c\right)\right)\right\}\right]g \tag{2}$$

$$(\rho c)_f\left[\frac{\partial T_f}{\partial t}+\mathbf{v}\cdot\nabla T_f\right]=k_f\nabla^2T_f+(\rho c)_p\left[D_B\nabla\varphi\cdot\nabla T_f+D_T\frac{\nabla T_f\cdot\nabla T_f}{T_f}\right] \tag{3}$$

$$\frac{\partial\varphi}{\partial t}+\mathbf{v}\cdot\nabla\varphi=D_B\nabla^2\varphi+\frac{D_T}{T_c}\nabla^2T_f \tag{4}$$

where $\mathbf{v}=(u,v,w)$ is the fluid velocity. Assuming the temperature to be constant and thermophoretic nanoparticles flux to be zero at the stress-free boundaries [12], the boundary conditions on T and φ shall be:

$$w=0,\ \ T=T_h,\ D_B\frac{\partial\varphi}{\partial z}+\frac{D_T}{T_c}\frac{\partial T}{\partial z}=0 \text{ at } z=0, \tag{5}$$

$$w=0,\ \ T=T_c,\ \ D_B\frac{\partial\varphi}{\partial z}+\frac{D_T}{T_c}\frac{\partial T}{\partial z}=0 \text{ at } z=d, \tag{6}$$

The present nanofluid model has been taken from earlier literature presented by Buongiorno, in which it was assumed that nanofluid is dilute mixture of nanoparticles i.e. $\varphi \ll 1$. Therefore, due to low concentration of nanoparticle, momentum equation for nanofluid remains same as for base fluid. Later, Nield and Kuznetsov [10] introduced/modified the buoyancy term for natural convection problem as weighted sum of densities of base fluid and nanoparticles with Boussinesq approximation to the base fluid density. The other conservative equations (mass, energy) contain the effect of slip mechanisms (Brownian motion and thermophoresis) due to nanoparticle migration.

For non-dimensionalizing the variables we take

$$(x^*,y^*,z^*)=(x,y,z)/d,\ t^*=t\alpha_f\Big/d^2,\alpha_f=\frac{k_f}{(\rho c)_f}$$