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Thermal transport characteristics pertinent to electrokinetic flow of power-law fluids in rectangular microchannels

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ABSTRACT

In the present study, the thermal characteristics of electroosmotic flow of power-law fluids in rectangular microchannels in the presence of pressure gradient are investigated. The governing equations for fully developed flow under H1 thermal boundary conditions are first made dimensionless and subsequently solved through a finite difference procedure for a non-uniform grid. The influence of the major parameters on thermal features of the flow such as the temperature distribution and Nusselt number is discussed by a complete parametric study. The results reveal that the channel aspect ratio and the non-Newtonian characteristic of the fluid can affect the thermal behavior of the flow. It is observed that decreasing the channel aspect ratio causes the energy generated due to the viscous heating to become more significant. Furthermore, the viscous dissipation is higher for shear-thickening fluids. The Nusselt number is ascertained to be an increasing function of the channel aspect ratio regardless of the flow behavior index and pressure gradient. Moreover, the results suggest that increasing the flow behavior index can either increase or decrease the Nusselt number, depending on the thermal conditions of the flow.

1. Introduction

Owing to the rapid development of microfluidic devices, several techniques have emerged for fluid delivery in these fluidic systems. Among the various techniques such as magnetohydrodynamics, piezoelectrics and electrohydrodynamics [1–3], electroosmosis [4] has been favored due to its many advantages over other types of pumping methods. Electroosmotic pumping systems are bidirectional and can generate continuous and pulse free flows. The precise flow control can be achieved by controlling the external electric field as well. In addition, they have no moving part and enjoy the merit of a simpler design and an easier fabrication.

Electroosmosis refers to flow generation through applying an electric field to an ionized solution in contact with charged surfaces. When exposed to an aqueous electrolyte solution, most surfaces obtain electrical charge. The interaction of charged surface with ions in electrolyte medium develops a layer named the electric double layer (EDL) which is shown schematically in Fig. 1. Within EDL, there is an excess of counter-ions over co-ions to neutralize the surface charge. If an electric field is applied tangentially along the surface, an electric body force will be exerted on the excess of counter-ions in EDL. This force will result in a net migration of the ions and because of viscous drag, the liquid is drawn by the ions; therefore, it flows through the channel. The electroosmotic flow velocity profile depends on the ratio of the characteristic length of the microchannel to the Debye length λ_D, where the Debye length characterizes the EDL thickness. When the Debye length is much smaller than the channel length scale, the velocity profile is nearly uniform with a large velocity gradient near the wall. On the other hand, when the Debye length is comparable with the channel characteristic length, the velocity profile is like that of classical pressure-driven flow.

The Primary researches attending to the electroosmotic flows were conducted by Burgreen and Nakache [5] and Rice and Whitehead [6] and since then a myriad of studies have been undertaken in order to investigate the different aspects of these flows. Owing to the electrical resistivity of fluids, applying an electric field to the solution will result in heat generation named Joule heating. Since one of the major applications of electroosmotic flow is in lab-on-chip devices, which concern thermally labile samples, the thermal analysis of electroosmotic flow is of considerable importance. Maynes and Webb [7] accomplished one of the earliest studies on thermal features of electroosmotic flow. In their study, the thermally fully developed, electroosmotically generated

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Nomenclature

c_p	specific heat at constant pressure [kJ kg^{-1} K^{-1}]
D_h	hydraulic diameter of channel [$=4HW/H+W$]
e	proton charge [C]
E_J	total Joule heating per channel length [W m^{-1}]
E_v	total viscous heating per channel length [W m^{-1}]
E_x	electric field in the axial direction [V m^{-1}]
F	component of body force vector [N m^{-3}]
$\mathbf{F}$	body force vector [N m^{-3}]
h	heat transfer coefficient [W m^{-2} K^{-1}]
H	half channel height [m]
k	Thermal conductivity [W m^{-1} K^{-1}]
k_B	Boltzmann constant [J K^{-1}]
m	flow consistency index [Pa s^n]
n	low behavior index
n_0	ion density at neutral conditions [m^{-3}]
Nu	Nusselt number [Eq. (38)]
p	pressure [Pa]
P	perimeter [m]
q	wall heat flux [W m^{-2}]
s	volumetric heat generation due to Joule heating [W m^{-3}]
S	dimensionless Joule heating parameter [Eq. (35)
S_v	dimensionless viscous dissipation parameter [Eq. (35)]
t	time [s]
T	absolute temperature [K]
u	axial velocity [m s^{-1}]
u_{HS}	Helmholtz–Smoluchowski velocity [Eq. (20)]
u_{PD}	pressure driven velocity [Eq. (24)]
$\mathbf{u}$	velocity vector [m s^{-1}]
W	half channel width [m]
x, y, z	coordinates [m]
Z	valence number of ions in solution

Greek symbols

α	channel aspect ratio [$=W/H$]
β	stretching parameter [Eqs. (40) and (41)]
Γ	velocity scale ratio [Eq. (23)]
$\dot{\gamma}$	magnitude of the strain rate tensor [s^{-1}]
$\dot{\boldsymbol{\gamma}}$	strain rate tensor [s^{-1}]
ε	fluid permittivity [CV^{-1} m^{-1}]
ζ	zeta potential [V]
θ	dimensionless temperature [Eq. (31)]
K	dimensionless Debye–Hückel parameter [$=H/\lambda_D$]
λ_D	Debye length [m]
μ	effective viscosity [Pa s]
ρ	density [kg m^{-3}]
ρ_e	net electric charge density [C m^{-3}]
σ	liquid electrical resistivity [Ω m]
σ_0	liquid electrical resistivity at neutral conditions [Ω m]
τ	stress tensor component [Pa]
$\boldsymbol{\tau}$	stress tensor [Pa]
φ	electrostatic potential [V]
Φ	externally imposed electrostatic potential [V]
ψ	EDL potential [V]

Subscripts

av	average
m	mean
w	wall

Superscripts

g	guess value
*	dimensionless variable
∧	transformed variable

convective transport has been analyzed for a parallel plate microchannel and circular microtube under imposed constant wall heat flux and constant wall temperature boundary conditions. The exact solutions for the dimensionless temperature profile and corresponding Nusselt number have been determined for both geometries and both thermal boundary conditions. Liechty et al. [8] extended the above study to high zeta potentials. Chen [9] investigated the thermal transport characteristics of combined electroosmotically and pressure driven flow in a slit microchannel. The mentioned study was extended to account for viscous heating effects by Sadeghi and Saidi [10], Dey et al. [11], and Chen et al. [12]. Sadeghi et al. [13] developed a variational-based semi analytical formulation for thermally developing electroosmotic flow in a rectangular microchannel. In a recent study, Matin and Khan [14] conducted an entropy generation analysis of heat and mass transfer in mixed electrokinetically and pressure driven flow.

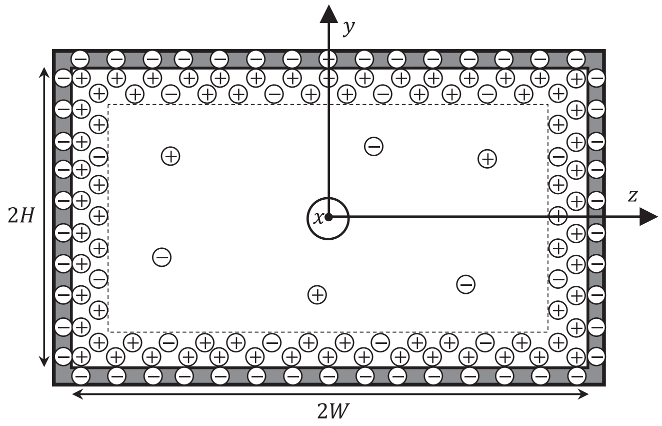

Fig. 1. Schematic of the physical problem along with the coordinate system; EDLs are the regions between the dashed lines and the channel wall.

Since the lab-on-chip devices are usually encountered with non-Newtonian behavior of working fluids, investigating the behavior of such fluids under the influence of the electroosmotic force is highly necessary for accurate design of these systems. The available literature indicates a multitude of researchers have recently focused on non-Newtonian fluid behavior in electrokinetically driven flows. Chakraborty [15] examined electroosmotic flow of power-law fluids in microchannels by means of a semi-analytical mathematical model. Zhao et al. [16] obtained an approximate analytical solution based on Debye–Hückel approximation for velocity profile of power-law fluids in a slit. Tang et al. [17] reported a numerical study on the electroosmotic flow of power-law fluids in slit microchannels employing the lattice Boltzmann method. Vasu and De [18] analyzed the electroosmotic flow of power-law fluids in a slit microchannel at high zeta potentials. Babaie et al. [19] numerically investigated the combined electroosmotically and pressure driven flow of power-law fluids in slit microchannels. Using the same rheological model, Shamshiri et al. [20] and Cho et al. [21] explored electrokinetically driven non-Newtonian fluid flow in annulus and wavy microchannels, respectively. Vakili et al. [22] performed a numerical