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Transient growth of acoustical energy associated with mitigating thermoacoustic oscillations

Xinyan Li [a], Dan Zhao [a,*], Xinglin Yang [b], Huabing Wen [b], Xiao Jin [b], Shen Li [b], He Zhao [b], Changqing Xie [c], Haili Liu [c]

[a] School of Mechanical and Aerospace Engineering, College of Engineering, Nanyang Technological University, Singapore 639798, Republic of Singapore
[b] School of Energy and Power Engineering, Jiangsu University of Science and Technology, Zhenjiang City, Mengxi Road 2, Jiangsu Province 212003, China
[c] Department of Energy and Electrical Engineering, Hunan University of Humanities Sciences and Technology, 410082 Loudi City, Hunan Province, China

HIGHLIGHTS

- A thermoacoustic model with a linearly varied mean temperature is developed.
- Pseudospectra analysis shows that the thermoacousic system is non-normal.
- Heat-driven sound energy is defined and calculated.
- Implementing LQG controller results in 76 dB sound pressure level reduction.
- Transient energy growth is associated with mitigating thermoacoustic oscillations.

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ABSTRACT

Energy conversion from heat to sound is desirable in some practical applications such as thermoacoustic heat engines or cooling systems. However, it is unwanted in gas turbine or aeroengine combustors. In this work, a Rijke-type thermoacoustic model with a linearly varied mean temperature configuration is developed. An acoustically compact heat source is confined and characterized by a modified form of King's law. Unlike previous models available in the literature, the mean temperature is assumed to undergo not only a sudden jump across the heat source but also linearly increasing and decreasing in the pre- and after-heating regions respectively. Such mean temperature configuration is consistent with the experimental measurement. Coupling the heat source model with a Galerkin series expansion of the acoustic fluctuations provides a platform to gain insight on (1) the nonlinearity of the thermoacoustic system, (2) onset of limit cycle oscillations, (3) predicting its non-normality behaviors, (4) energy distribution and transfer between neighboring eigenmodes, and (5) evaluating the performance of feedback controllers. Pseudospectra and transient energy growth analyses are then performed. It reveals that the system is non-normal. And it is associated with transient growth of acoustical energy. The non-normality is found to be less intensified in comparison with that in a system with an invariant mean temperature from pre- and after-heating regions. To mitigate these limit cycle oscillations, the heat-to-sound coupling is interrupted by implementing multiple monopole-like actuators driven by a LQG (Linear Quadratic Gaussian) controller. For comparison, a pole-placement controller is also implemented. Approximately 76 dB sound pressure level reduction is achieved. However, implementing the LQG controller is shown to be associated with transient growth of acoustical energy, which has potential to trigger thermoacoustic instability. The present work opens up new applicable way to model thermoacoustic systems in the presence of a mean temperature gradient. Furthermore, it reveals new potential risk of applying active controllers to stabilize thermoacoustic systems.

1. Introduction

In order to achieve low NOx emissions, combustion systems in ground-based gas turbines and aero-engines [1] tend to operate under lean premixed condition to meet stringent emission require-

* Corresponding author.
E-mail address: zhaodan@ntu.edu.sg (D. Zhao).

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Nomenclature

$\mathbf{A}$	coefficient matrix of the linearized governing equation
$\bar{c}$	speed of sound in air, m/s
C_p, C_v	heat capacity ratio at constant pressure and volume, kJ/kg K
d_w	the diameter of the heated wires, m
E_1, E_2	acoustical energy density at ω_1 and ω_2, W/m^2
E_s	total acoustical energy density, W/m^2
G	transient growth of acoustical energy
J_0	the Bessel function of order zero
$\mathcal{K}$	control gain
L	the combustor length, m
L_w	the length of the heated wires, m
m_1, m_2	the mean temperature gradient, K/m
$\overline{M}$	mean flow Mach number
N	the number of actuators
$\mathcal{N}$	interaction index
N_1	the number of eigenmodes
p	instantaneous pressure, Pa
$\bar{p}$, p'	the mean and fluctuating pressure, Pa
Q'_s	unsteady heat release rate, kJ/s
$\mathcal{Q}'_a$	monopole-like sound source, kJ/s^2
R_g	the gas constant, J/mol K
$\mathcal{S}$	cross-sectional area of the tube, m^2
S_k, R_k	actuation coefficients
T_w	the temperature of the heated wires, K
$\overline{T}_{1f}$, $\overline{T}_{2f}$	initial mean temperatures in pre- and after-heating regions, K
$\overline{T}$	the mean temperature, K
u	instantaneous flow velocity, m/s
$\mathbf{u}$	control input
$\bar{u}$, u'	the mean and fluctuating velocity, m/s
$\mathbf{w}(t)$, $v(t)$	white Gaussian noise
x_f	axial location of the heat source, m
x_{ak}	axial location of the kth actuator, m
x_s	axial location of the sensor, m
$\mathbf{y}$	vector of the state variables
Y_0	the Neumann function of order zero

Greek letters

α_{ak}	the ratio of the cross-section area
δ	the Dirac delta function
γ	the ratio of specific heat
κ_m	the normalized coefficient for the mth mode
λ	conduction coefficient, W/mK
ω	oscillation frequency, rad/s
ψ_m	basis function
$\bar{\rho}$	mean air density, kg/m^3
v_{ak}	the actuation signal
τ_f	time delay, s
ξ_m	the damping coefficient

Subscript

a	actuator
m	mth mode
k	kth actuator
1	before-heating
2	after-heating
f	heat source

Superscript

$'$	fluctuating value
$\dot{}$	time derivative
$\bar{}$	mean value
$\hat{}$	approximation value

ments. However, under such lean premixed condition gas turbine combustors are more prone to thermoacoustic instability [1,2]. Such instability is generally caused by the coupling between unsteady combustion and acoustic disturbances. Unsteady heat release, which is an efficient monopole-like sound source, produces acoustic waves. When the sound waves propagate within the combustor, part of them are reflected from boundaries due to the impedance change, and return back to the combustion zone and further perturb the combustion process. If unsteady heat is added in phase with the pressure oscillations [3–5], acoustical energy is increased and finally 'saturated'. And limit cycle thermoacoustic oscillations are generated.

Such large-amplitude flow oscillations are wanted in thermoacoustic engines or cooling systems [6–9]. Efficient heat-to-sound conversion is desirable in some practical applications. When unsteady heat generates acoustic sound during the process of transferring from a high temperature source to a low-temperature one, this sound could be used to produce electric power via piezoelectric generator [8] or linear alternator [4]. A variety of heat sources [8], such as solar energy and industrial waste heat, can be utilized. Thermoacoustic engines eliminate all the moving parts and are 'pistonless' [10]. The working gas undergoes compression and expansion processes in the form of sound waves to achieve the conversion of thermal energy to mechanical work [7,9]. So thermoacoustic engines can achieve high reliability and durability. There are two general types. One is a standing-wave one [8] and the other is traveling-wave one [9,7]. One typical example of standing-wave engines is Rijke-type thermoacoustic system [11]. It is a straight vertical tube with a mean flow and a heat source placed in its lower half. Under certain conditions,the interaction between unsteady heat release and acoustic disturbances in the tube may give rise to self-sustained thermoacoustic oscillations. However, this heat-to-sound conversion (also known as thermoacoustic instability) is undesirable in gas turbines and aero-engines, since large-amplitude flow oscillations may result in unacceptable noise and structural vibration, even catastrophic engine failure.

Stable operation is one of the main requirements for gas turbine combustors. Thus it is important to gain insight on the onset of thermoacoustic instability [12,13,2] and to develop mitigation/-control approaches. Over last few decades, thermoacoustic instability has been intensively studied [14–20]. Generally, linear modal analysis is conducted via calculating the eigenfrequencies of thermoacoustic systems [3]. However, when the thermoacoustic eigenmodes are not orthogonal, the system is non-normal. And acoustic fluctuations may undergo transient growth. If the transient growth is large enough (in comparison with acoustic losses and damping), thermoacoustic instability might be triggered.

To gain insight on the finite-time (transient) stability behaviors, transient growth analysis of acoustical energy in a thermoacoustic system receives more attention recently [16–18]. A choked thermoacoustic combustor is studied to gain insight on its non-normality. It is found that the maximum transient growth can be 10^3 times greater than the initial disturbances energy. Experimental measurement of the transient growth was conducted on a premixed gas turbine combustor [15]. The combustor inlet is not acoustically open or closed. Theoretical non-normality investigations are conducted on modeled thermoacoustic systems