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Mathematical modelling of heat transfer effects on swallowing dynamics of viscoelastic food bolus through the human oesophagus

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ABSTRACT

The present paper describes a mathematical study on peristaltic flow of viscoelastic fluids (with the robust Jeffrey model) through a finite length channel under the influence of heat transfer. The study is motivated by the need to further elucidate the mechanisms inherent in swallowing of diverse food bolus types (bread, fruit jam and almost all edible semi-solids) through the oesophagus, by taking account of the viscous and elastic effects. The expressions for temperature field, axial velocity, transverse velocity, volume flow rate, pressure gradient, local wall shear stress, mechanical efficiency, stream function, and reflux limit are obtained, when the Reynolds number is small and the wavelength is large, by using appropriate analytical and numerical methods. The computational results are presented in graphical form. The influence of thermophysical (heat transfer), relaxation time and retardation time parameters on pressure distribution, local wall shear stress profiles, temperature profiles and velocity profiles are studied in detail. Furthermore we investigate the effects of these parameters on two inherent phenomena (reflux and trapping) characterizing peristaltic flow using streamline plots. The present study emphasizes an important observation, namely that pressure along the entire length of the channel reduces when the magnitude of relaxation time (retardation time is fixed) or Grashof number or indeed thermal conductivity increase, whereas pressure is enhanced by increasing the magnitude of retardation time (relaxation time is fixed).

1. Introduction

The oesophagus is a muscular tube in the chest that connects the mouth and throat to the stomach. Muscles in the wall of the oesophagus work in a coordinated manner to push food and liquids down into the stomach. This muscular movement is termed peristalsis. Peristalsis is a natural mechanism of pumping that is observed in the case of most physiological fluids. This behavior is usually associated with a progressive wave of area contraction or expansion along the length of the boundary of a fluid-filled distensible tube. This mechanism takes place in many practical appliances including roller and finger pumps, heart-lung machines, blood pump machines, dialysis machines and also transport of noxious fluids in the nuclear and chemical waste industries. Owing to this diverse range of applications, in recent years, mathematical modelling of peristaltic movement has received increasing interest among scientific researchers. Historically peristaltic fluid dynamics studies were initiated by Shapiro et al. [1] who theoretically examined the peristaltic flow of viscous fluid induced by sinusoidal wall propagation. They performed the analysis under long wavelength and low Reynolds number assumptions and discussed the phenomena of reflux and trapping during peristalsis. Later several workers, besides these pioneers, enriched the knowledge and information on this topic with valuable contributions.

Heat transfer techniques change the internal energy of both systems involved and follow the first law of thermodynamics. Heat is generally transferred from objects of varying temperatures via conduction and convection. Several processes rely on heat transfer and the corresponding thermal coefficients. These include distillation, crystallization, and other boiling operations. Evaporation is also a common chemical application that involves concentrating a solution containing a non-volatile solute by boiling away the solvent. This process is implemented for the production of orange juice concentrate, concentrated H_2SO_4, and production of distilled water. In physiology, it is used to study the properties of tissues. Recent advances in the application of heat (hyperthermia), radiation (laser therapy), and coldness (cryosurgery), as means to destroy undesirable tissues, such as cancer, have stimulated much

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interest in the study of thermal modelling in tissue. In gastro-intestinal transport, heat also plays an important role since the bolus transported will behave differently under different thermal conditions linked to the temperature of the bolus, body temperature, etc. Several authors [2–16] have therefore reported the influence of heat transfer on peristaltic flow of Newtonian and non-Newtonian fluids (Reiner Rivlin fluid, Jeffrey-six constant fluid, second grade fluid, third order fluid, fourth grade fluid, Herschel Bulkley fluid and Johnson Segalman fluid) with or without the effect of magnetic field through uniform/non-uniform/asymmetric channels/vertical annulus systems and also porous media. These studies discussed the characteristics of physical parameters on flow behavior and identified interesting thermal effects on bolus development. Further interesting studies include the recent papers by Abd elmaboud and Mekheimer [17] which considered thermal transport in transient flow in a vertical constricted annulus. Mekheimer et al. [18] studied peristaltic hydromagnetic visco-elastic heat and mass transfer with magnetic induction effects. Abd elmaboud and Mekheimer [19] investigated peristaltic second-order fluid transport in porous media. Also Mekheimer and Abdel-Waheb [20] discussed the net annulus peristaltic flow of compressible fluids.

Viscoelastic fluids play important role in modern industrial and also biophysical processes. They are non-Newtonian fluids, which exhibit simultaneously both viscous and elastic both properties. Popular basic models for viscoelastic materials are the Maxwell model (for fluids) and Kelvin-Voigt model (for solids). The Jeffrey model (for semi-solids) constitutes a robust viscoelastic formulation which falls between the Maxwell and Kelvin-Voigt models. Bread, fruit jam and almost all edible semi-solids bear both viscous and elastic properties which can be simulated using the Jeffrey viscoelastic model. Pandey and Tripathi [21] and Kothandapani and Srinivas [22] have described simulations of peristaltic flows of viscoelastic fluids with the Jeffrey model. They have discussed the effect of relaxation and retardation time on peristaltic transport characteristics.

The studies alluded to above, on peristaltic transport of Newtonian and non-Newtonian fluids have all been confined to the case of an infinite length tube, ignoring the inherently non-steady effects associated with finite length tubes encountered in real peristaltic pumps and actual biological vessels. The first significant study of peristaltic flow of a Newtonian fluid for finite length tubes was presented by Li and Brasseur [23], who focused on both local and global dynamics aspects of transport. Brasseur and Dodds [24] addressed the issue of local dynamics such as spatial-temporal variations in local stresses in terms of the motility and efficacy of the transport process. Their results are found to correlate well with the experimental findings of Dodds [25]. Misra and Pandey [26] extended these Newtonian finite length tube studies to the case of power-law fluids. Several investigations on peristaltic flow of non-Newtonian fluids have also been recently reported for finite length channels/tubes by Pandey and Tripathi [21,27–34]. Tripathi [35,36] has further improved these models for viscous fluids with exponential variable viscosity and with heat transfer. Other interesting studies [37–42] on physiological flows are recently reported in the literature of bio-fluid-mechanics.

Motivated by the investigations given above and the increasing need to refine understanding of the complex flow dynamics in multi-physical peristaltic transport processes, the present article investigates the peristaltic flow of a Jeffrey viscoelastic fluid through a finite length channel with the effect of heat transfer. The present problem is analyzed using lubrication theory. The analysis and the results of the study are particularly applicable to the swallowing of food bolus through the oesophagus and the transportation of a wide range of actual foods being conveyed via the digestive tract.

2. Mathematical model

The constitutive equation [26] for the wall geometry studied, relating to the two-dimensional channel model of the oesophagus (as shown in Fig. 1) due to propagation of train waves is given by:

$$\tilde{h}\left(\tilde{\xi},\tilde{t}\right) = a - \tilde{\phi}\cos^2\frac{\pi}{\lambda}\left(\tilde{\xi}-c\tilde{t}\right) \tag{1}$$

where $\tilde{h},\tilde{\xi},\tilde{t},a,\tilde{\phi},\lambda$ and c represent the transverse vibration of the wall, the axial coordinate, time, the half width of the channel, the amplitude of the wave, the wavelength and the wave velocity respectively.

The equations governing the unsteady motion of an incompressible Jeffrey viscoelastic fluid with heat transfer through the channel can be stated as follows:

$$\left.\begin{aligned}
&\rho\left(\frac{\partial}{\partial\tilde{t}}+\tilde{u}\frac{\partial}{\partial\tilde{\xi}}+\tilde{v}\frac{\partial}{\partial\tilde{\eta}}\right)\tilde{u} = -\frac{\partial\tilde{p}}{\partial\tilde{\xi}}+\frac{\partial\tilde{S}_{\tilde{\xi}\tilde{\xi}}}{\partial\tilde{\xi}}+\frac{\partial\tilde{S}_{\tilde{\xi}\tilde{\eta}}}{\partial\tilde{\eta}}+\rho g\alpha(T-T_0),\\
&\rho\left(\frac{\partial}{\partial\tilde{t}}+\tilde{u}\frac{\partial}{\partial\tilde{\xi}}+\tilde{v}\frac{\partial}{\partial\tilde{\eta}}\right)\tilde{v} = -\frac{\partial\tilde{p}}{\partial\tilde{\eta}}+\frac{\partial\tilde{S}_{\tilde{\eta}\tilde{\eta}}}{\partial\tilde{\eta}}+\frac{\partial\tilde{S}_{\tilde{\eta}\tilde{\xi}}}{\partial\tilde{\xi}},\\
&\frac{\partial\tilde{u}}{\partial\tilde{\xi}}+\frac{\partial\tilde{v}}{\partial\tilde{\eta}}=0,\\
&\rho c_p\left(\frac{\partial}{\partial\tilde{t}}+\tilde{u}\frac{\partial}{\partial\tilde{\xi}}+\tilde{v}\frac{\partial}{\partial\tilde{\eta}}\right)T = k\left(\frac{\partial^2 T}{\partial\tilde{\xi}^2}+\frac{\partial^2 T}{\partial\tilde{\eta}^2}\right)+\Phi
\end{aligned}\right\} \tag{2}$$

where $\rho,\tilde{u},\tilde{v},\tilde{\eta},\tilde{p},g,\alpha,T,c_p,k,\Phi$ and $\tilde{S}_{\tilde{\xi}\tilde{\xi}},\tilde{S}_{\tilde{\xi}\tilde{\eta}},\tilde{S}_{\tilde{\eta}\tilde{\xi}},\tilde{S}_{\tilde{\eta}\tilde{\eta}}$ are the fluid density, axial velocity, transverse velocity, transverse coordinate, pressure, acceleration due to gravity, coefficient of linear thermal

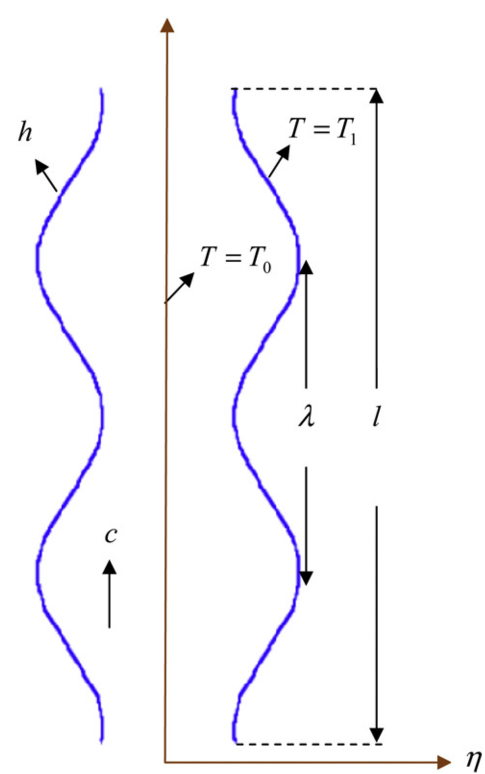

Fig. 1. Geometry of the oesophageal swallowing.