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Topology optimization of heat conduction paths by a non-constrained volume-of-solid function method

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ABSTRACT

A novel computational approach based on a non-constrained formulation with a volume-of-solid (VOS) function equation is firstly presented for topology design of heat conductive solid paths between constant-temperature objects. In the first step of the approach, the distributions of the VOS function and the temperature in the original design domain are carried out by simultaneously solving the VOS function equation and the heat conduction equation. Secondly, the shape outline of the heat conduction path leading to a maximum heat transfer rate per unit solid mass is determined by selecting a cut-off value of the VOS function. Performance of this approach is tested for three two-dimensional test cases. Various thermal boundary configurations are taken into consideration to demonstrate the validity of the present method. Results show that the present computational method is capable of predicting the optimal shapes of the heat conduction paths for the test cases efficiently.

1. Introduction

Heat dissipating devices have been widely applied in numerous engineering applications. Therefore, demand of the performance of heat conductive materials used in the heat dissipating devices becomes more critical. Compared to other solid materials, metals have been widely known for their superior heat transfer characteristics. However, these characteristics come at the cost of weight, coefficient of thermal expansion mismatch and freedom of design due to manufacturing constraints. Hence, the desire for thermally conductive plastics is constantly growing. By applying fillers like highly heat-conductive pitch-based carbon fibers, ultra-high thermal conductivity polymer composites have been developed. Using mats of vapor-grown carbon fibers impregnated with epoxy, values of thermal conductivity up to 300 to 660 W m^{-1} K^{-1} were reported [1]. Thermally conductive plastics have numerous properties that make them superior to other materials in many applications, such as resistance to corrosion and chemicals, high strength-to-weight ratio, low toxicity, etc. Moreover, plastics generally have the advantage of easy manufacturing by using rapid prototyping or 3D printing technology, especially for the parts of complex shapes. Recently, an essential issue is how to design the structures with a rational distribution of thermally conductive plastics so as to increase the heat transfer rate but reduce the mass of material utilized. In other word, the issue is to find an optimal shape of a heat conduction path that leads to a maximum heat transfer rate per unit solid mass.

In most of the cases, the determination of the shape of the heat transfer path between constant-temperature objects is merely based on the experience of the designers or costly experimental information. It is usually carried out in a relatively time-consuming trial-and-error process. In this regard, an efficient computational method for topology design of the shape of the heat conduction path is definitely desired.

Topology optimization is a computational approach which optimizes material layout within a given design space, for given loads or boundary conditions, such that the designed layout may meet the desired performance requirement. Using topology optimization, engineers can find the best concept design at a relatively low cost in a short period of time. In the past several decades, structural mechanics topology optimization has been done through the use of a number of different techniques based on genetic algorithm [2], level set method [3], COC algorithm [4], and finite-element methods [5,6]. Applications of these topology optimization techniques cover a wide range of engineering problems, such as composite laminates [7], aircraft components [8], and so on.

In the field of heat conduction structures, the progress of the optimization methods has also been advanced recently. For example, Bejan and coworkers [9–12] developed a tree network for heat transfer based on a constructal theory which assumes that for a finite-size system to persist in time (to live), it must evolve in such a way that it provides greater access to the imposed currents that

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Nomenclature

B	object
k_{eff}	effective thermal conductivity, W m^{-1} K^{-1}
k_s	thermal conductivity of pure solid, W m^{-1} K^{-1}
L	side length of original design domain, m
m	mass of solid material, kg
n	power constant in effective thermal conductivity equation
N	normal coordinate to boundary of original design domain
Q	heat transfer rate, W
$\vec{q}$	heat flux vector, W m^{-2}
S_1	coefficient of Equation (3), K
S_2	coefficient of Equation (3), W m^{-1} K^{-1}
T	temperature in original design domain, K
T_s	pure solid temperature in conduction path, K
v_p	volume fraction of void
x, y	rectangular coordinates, m
Greek symbols	
α	boundary of constant-temperature object
ζ	volume-of-solid function
Ψ	orthogonality index, W m^{-3}
Subscripts	
i	index of constant-temperature object

flow through it. The evolution of design and pattern in nature is a phenomenon of physics. Guo, Cheng, and Xia [13] performed heat conduction structure optimization based on the least dissipation principle of heat transport potential capacity for several practical examples. Cheng and Chang [14,15] designed the shapes for cylinders and sliders to meet different required loading conditions based on the simplified conjugate-gradient method (SCGM) [16]. In addition, the topology optimization methods have been extended successfully toward optimal design of numerous heat conduction paths [17–21]. However, to the authors' knowledge, the topology optimization of heat conduction problems has been relatively less addressed as comparing with the structural mechanics problems in spite its potential significances. Therefore, there is still room in development of novel concept to accelerate the progress in topology optimization of the heat conduction structures.

Traditionally, the optimization tasks were performed by building a mathematical model which maximizes or minimizes an objective function subject to some certain constraints. A suitable thermal performance index must be selected as the objective function, which could be dissipation of heat transport potential capacity [13], highest temperature in solution domain [18,19], entropy production [22], and so on. In order to maximize or minimize the objective function, one of the crucial steps with the topology optimization is to evaluate the topological sensitivities of the objective function on the designed parameters.

It is noted that very few people have studied the shape of heat transfer path between constant-temperature objects or its applications. Iga et al. [23] mentioned one test example showing the conduction path between one heat-flux object and three other low-temperature objects; however, no further information were reported in their study.

In this study, a new approach is presented for topology design of heat conductive solid paths leading to maximum heat transfer rate per unit solid mass between constant-temperature objects. The approach is based on a non-constrained formulation with a volume-of-solid (VOS) function equation and the topology optimization of the shape of the heat conduction paths leading to maximum heat transfer rate per unit solid mass is carried out by simultaneously solving the partial-differential VOS function equation and the heat conduction equation. To demonstrate the validity of the present approach, three two-dimensional test cases are investigated in this study.

2. Topology design theory

In this study, the solid material distribution method is based on a volume-of-solid (VOS) function ζ, which represents the volume fraction of the solid material in each of the computational grid cells. The present topology optimization problem for heat conductive solid paths can be stated mathematically as follows,

$$\begin{aligned}&\text{Find}:\ \zeta(x,y),\ \text{with}\ \zeta\in[0,\ 1]\\&\text{Maximize}:\ Q/m\\&\text{Subject to}:\ T=T_i\ \text{on}\ \alpha_i\ \text{for}\ B_i,\ i=1,\ 2,\ 3,\ldots\end{aligned} \tag{1}$$

where Q represents heat transfer rate from the hot object (B_1) to the cold objects (B_i, $i = 2, 3,\ldots$); m the mass of solid material; T_i and α_i the temperature and the boundary of the i-th constant-temperature object, respectively. Schematic of the problem is illustrated in Fig. 1. The optimal design of the heat conduction path is performed by determining the solid material distribution $\zeta(x, y)$ leading to maximum heat transfer rate per unit solid mass. For pure solid material region, $\zeta = 1$ and for pure void region, $\zeta = 0$. As more solid material is invested in building the path, the pathway area is increased, and hence, heat transfer rate (Q) is usually elevated. However, questions that must be asked are: Do we get any benefit of the investment? Based on what heat transfer index we can fairly assess the heat transfer benefit? The heat transfer rate per unit solid mass used represents a utilization efficiency of the solid materials for heat transfer. In the present study, higher heat transfer rate per unit solid mass is pursued for improving the utilization efficiency of the solid materials. Therefore, the heat transfer index used here is Q/m rather than Q.

In some certain existing studies, for example, Ref. [19], the constraint of the optimization of heat conduction structure is given with a fixed amount of mass m. Using this constraint, one may obtain some interesting results. However, this constraint can only fix the amount of solid mass in use rather than increase the

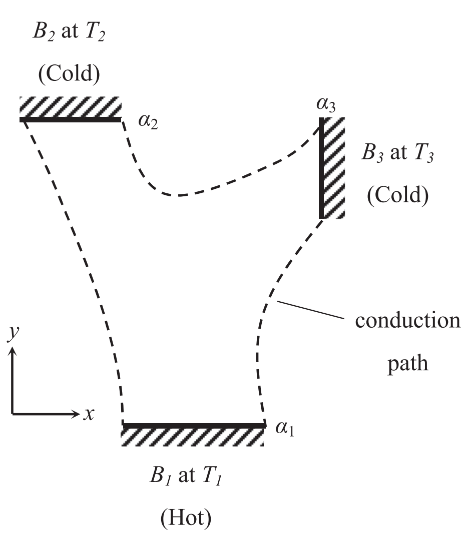

Fig. 1. Schematic of topology design of conduction path.