International Journal of Thermal Sciences 78 (2014) 90–100

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Optimized analytical solution for oblique flow of a Casson-nano fluid with convective boundary conditions

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ARTICLE INFO

Article history:
Received 15 July 2013
Received in revised form
30 November 2013
Accepted 1 December 2013
Available online 11 January 2014

Keywords:
Casson fluid
Nano fluid
Stagnation point
Stretching surface
Brownian motion
Thermophoresis
Heat transfer
Optimal homotopy analysis method

ABSTRACT

The present study deals with the steady stagnation point flow of a Casson nano fluid in the presence of convective boundary conditions. The fluid strikes the wall in an oblique manner. The governing nonlinear partial differential equations of the physical problem are presented and then converted into nonlinear ordinary differential equations by using similar and non-similar variables. The resulting ordinary differential equations are successfully solved analytically using Optimal Homotopy analysis method (OHAM) via BVPh2.0. Non-dimensional velocities, temperature and Nanoparticle concentration profiles are expressed through graphs. In order to understand the flow behavior at the stretching convective surface, numerical values of skin friction co-efficient and local heat and mass flux are tabulated. Comparison of the present analysis is made with the previous existing literature and an appreciable agreement in the values is observed for the limiting case.

1. Introduction

The pioneer work for the stagnation point flow was done by Hiemenz [1] for the 2-dimensional flow. Stagnation-point flow basically describes the fluid motion near the stagnation region of a solid surface exists in the case of fixed as well as moving body in a fluid. Stagnation point flow with various physical effects has great physical importance, for example in the prediction of skin-friction as well as heat/mass transfer near stagnation regions of bodies in high speed flows, design of thrust bearings and radial diffusers, drag reduction, transpiration cooling and thermal oil recovery. Similarity solutions for the three-dimensional flow and heat transfer of a power-law fluid near a stagnation point of an isothermal surface are presented by Subba et al. [2]. Similarity solution of the laminar boundary layer equations corresponding to an unsteady stretching surface has been studied by Elbashbeshy and Bazid [3]. They analyzed that the thermal boundary layer thickness decreases with unsteadiness parameter and Prandtl number while the momentum boundary layer thickness decreases with unsteadiness parameter. The effects of variable thermal conductivity and radiation on the flow and heat transfer of an electrically conducting micropolar fluid over a continuously stretching surface with varying temperature in the presence of a magnetic field are considered by Mahmoud [4]. He solved the resulting system of coupled non-linear ordinary differential equations numerically and showed that the thermal boundary thickness increases as the thermal conductivity parameter increases, while it decreases as the radiation parameter increases. The steady two-dimensional MHD stagnation point flow toward a stretching sheet with variable surface temperature is investigated by Ishak et al. [5]. They found that the heat transfer rate at the surface increases with the magnetic parameter when the free stream velocity exceeds the stretching velocity. Very recently Akbar et al. [6] discussed numerical solutions of Tangent Hyperbolic fluid toward a stretching sheet with MHD.

Heat transfer under convective boundary conditions plays a vital role in processes such as thermal energy storage, gas turbines, nuclear plants etc. Yao et al. [7] discussed the flow and heat transfer in a viscous fluid flow over a stretching/shrinking sheet with convective boundary conditions. They found that the convective boundary conditions result in temperature slip at the wall and this temperature slip is greatly affected by the mass transfer parameter, the Prandtl number, and the wall stretching/shrinking parameters. Gbadeyan et al. [8] investigated the boundary layer flow of a nano

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http://dx.doi.org/10.1016/j.ijthermalsci.2013.12.001

fluid past a stretching sheet with a convective boundary condition in the presence of magnetic field and thermal radiation. They found out that the local concentration of nanoparticles increases as the convective Biot number increases but decreases as the Lewis number increases. Aziz [9] examined a similarity solution for laminar thermal boundary layer over a flat surface with convective surface boundary condition. Similarity solutions for flow and heat transfer analysis over a permeable surface in connection with convective boundary condition have been discussed by Ishak [10]. Influence of buoyancy forces and thermal boundary layer over a vertical plate with a convective surface boundary condition has been developed by Makinde and Olanrewaju [11]. In another article Aziz [12] studied the hydrodynamic and thermal slip boundary layers flow over a flat plate with constant heat flux boundary condition. Some recent works concerning the convective boundary conditions are cited in Refs. [13–15].

A nanoparticle is a microscopic particle with at least one dimension less than 100 nm. Nanoparticle research is currently an area of intense scientific interest due to a wide variety of potential applications in biomedical, optical and electronic fields. The Nano fluid model was first developed by Choi [16]. After Choi this useful area has been recently highlighted by Buongiorno [17], Khanafer et al. [18], Das et al. [19], Kuznetsov et al. [20], Akbar and Nadeem [21], Nadeem et al. [22], Pop et al. [23] in their research work.

The main goal of the present study is to find the analytical simulation for the problem of oblique stagnation point flow of a non-Newtonian fluid (Casson model) under convective boundary conditions and we will extend the optimal homotopy analysis method for it. In this way our research paper has been organized as follow. The flow problem is formulated in Section 2. Sections 3 and 4 deal with the series solution and their convergence respectively, utilizing the application of the OHAM to construct the approximate solutions for the governing equations. Results and discussions are presented in Section 5. The conclusions are summarized in the last section of manuscript. Optimal Homotopy analysis method proposed by Liao [25] is used in computation of series solutions. Several edifying articles concerning the study may be found in Refs. [26–31]. At the end, the physical behaviors of pertinent parameters have been discussed.

2. Mathematical formulation

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid can be written as [24]

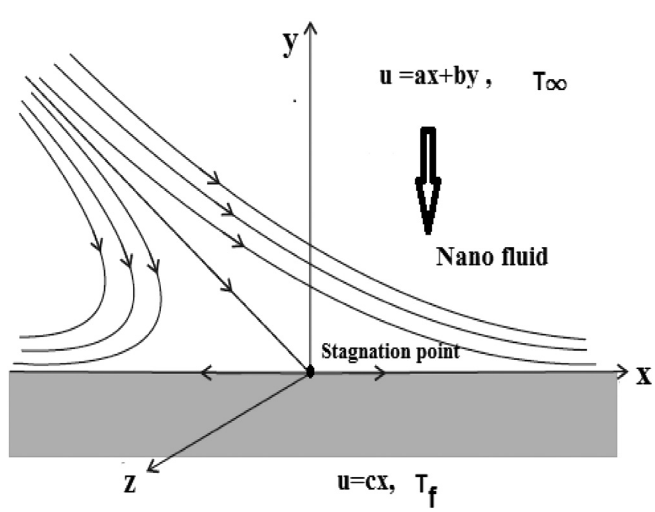

Fig. 1. A physical description of the problem.

$$\tau_{ij} = -p\delta_{ij} + \begin{Bmatrix} 2\left[\mu_B + \frac{p_y}{\sqrt{2\pi}}\right]e_{ij}, & \pi > \pi_c \\ 2\left[\mu_B + \frac{p_y}{\sqrt{2\pi_c}}\right]e_{ij}, & \pi_c > \pi \end{Bmatrix}, \tag{1}$$

where $\pi = e_{ij}e_{ij}$ and e_{ij} is the (i,j)th component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product, μ_B is plastic dynamic viscosity of the non-Newtonian fluid, p_y is yield stress of slurry fluid.

Consider the steady two-dimensional stagnation point flow of a Casson-nano fluid over a stretching surface. Two equal and opposite forces are applied along the x-axis so that the surface is stretched keeping the origin fixed, as shown in Fig. 1. We further assume that the surface has convective fluid temperature T_f and uniform ambient temperature T_∞. Here $(T_f > T_\infty)$.

The governing equations of the flow are defined as

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0, \tag{2}$$

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} + \frac{1}{\rho_f}\frac{\partial p^*}{\partial x^*} = (\nu)\left(1+\frac{1}{\beta}\right)\nabla^{*2}u^*, \tag{3}$$

$$u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} + \frac{1}{\rho_f}\frac{\partial p^*}{\partial y^*} = (\nu)\left(1+\frac{1}{\beta}\right)\nabla^{*2}v^*, \tag{4}$$

$$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \alpha^*\nabla^{*2}T^* + \frac{(\rho c)_p}{(\rho c)_f}\left[D_B\nabla^*C^*\cdot\nabla^*T^* + \frac{D_T}{T_\infty}\nabla^*T^*\cdot\nabla^*T^*\right], \tag{5}$$

$$\left[u^*\frac{\partial C^*}{\partial x^*} + v^*\frac{\partial C^*}{\partial y^*}\right] = D_B\nabla^{*2}C^* + \frac{D_T}{T_\infty}\nabla^{*2}T^*, \tag{6}$$

with the following conditions [23]

$$\begin{aligned} &u^* = cx^*,\ v^* = 0,\ -k\tfrac{\partial T^*}{\partial y^*} = h\left(T_f - T^*\right),\ C^* = C_w \ \text{at}\ y^* = 0, \\ &u^* = ax^* + by^*,\ T^* = T_\infty,\ C^* = C_\infty \ \text{as}\ y^* \to \infty, \end{aligned} \tag{7}$$

where a, b, and c are positive constants with dimensions of inverse time.

In above expressions u^* and v^* are the velocity components along the x^*- and y^*-axes, respectively, ρ_f the fluid density, ρ_p the nanoparticle mass density, ν is the kinematic viscosity, T is the temperature, c_p the specific heat of the material, α^* the thermal diffusivity of the fluid, k is the thermal conductivity of fluid, and h is the convective heat transfer coefficient.

Introducing the following quantities

$$\begin{aligned} &x = x^*\sqrt{\tfrac{c}{\nu}},\ y = y^*\sqrt{\tfrac{c}{\nu}},\ u = \tfrac{1}{\sqrt{\nu c}}u^*,\ v = \tfrac{1}{\sqrt{\nu c}}v^*, \\ &p = \tfrac{1}{\mu c}p^*,\ T = \tfrac{T^*-T_\infty}{T_f-T_\infty},\ C = \tfrac{C^*-C_\infty}{C_w-C_\infty}. \end{aligned} \tag{8}$$

Eqs. (2)–(6), become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{9}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \left(1+\frac{1}{\beta}\right)\nabla^2 u, \tag{10}$$