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# Harmonic analysis in integrated energy system based on compressed sensing

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## HIGHLIGHTS

- We propose a harmonic/inter-harmonic analysis scheme with compressed sensing theory.
- Property of sparseness of harmonic signal in electrical power system is proved.
- The ratio formula of fundamental and harmonic components sparsity is presented.
- **S**pectral **P**rojected **G**radient-**F**undamental **F**ilter reconstruction algorithm is proposed.
- **SPG-FF** enhances the precision of harmonic detection and signal reconstruction.

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## ABSTRACT

The advent of *Integrated Energy Systems* enabled various distributed energy to access the system through different power electronic devices. The development of this has made the harmonic environment more complex. It needs low complexity and high precision of harmonic detection and analysis methods to improve power quality. To solve the shortages of large data storage capacities and high complexity of compression in sampling under the Nyquist sampling framework, this research paper presents a harmonic analysis scheme based on compressed sensing theory. The proposed scheme enables the performance of the functions of compressive sampling, signal reconstruction and harmonic detection simultaneously. In the proposed scheme, the sparsity of the harmonic signals in the base of the Discrete Fourier Transform (DFT) is numerically calculated first. This is followed by providing a proof of the matching satisfaction of the necessary conditions for compressed sensing. The binary sparse measurement is then leveraged to reduce the storage space in the sampling unit in the proposed scheme. In the recovery process, the scheme proposed a novel reconstruction algorithm called the Spectral Projected Gradient with Fundamental Filter (SPG-FF) algorithm to enhance the reconstruction precision. One of the actual microgrid systems is used as simulation example. The results of the experiment shows that the proposed scheme effectively enhances the precision of harmonic and inter-harmonic detection with low computing complexity, and has good capability of signal reconstruction. The maximum detection error reaches 0.0315%, and the reconstruction signals to noise ratio (SNR) is higher than 89 dB.


## 1. Introduction

In recent years, based on rising attention for environmental protection on a worldwide basis, distributed renewable energy gradually replaces conventional fossil fuel in many areas. When photovoltaic system, wind turbine and electric vehicles are connected to the grid, nonlinear power electronic devices bring serious power quality problems [1–3]. Lv, considering large-scale integration of renewable energy resources, researched the interactive energy management operation to improve the power quality [1]. Boynuegri et al. [3] studied a novel power conditioning unit to decreasing voltage distortion caused by electric vehicles. To solve the power quality problems in Integrated Energy Systems, methods with low complexity and high precision of harmonic detection and analysis have become the key technique of improving power quality in Integrated Energy Systems, especially in microgrid. Several techniques and models have been developed in recent years to address power quality issues. The methods based on wavelet packet transform [4], hanning window [5], minimize sidelobe windows [6], discrete Fourier transform (DFT) [7], as well as

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FFT and its improved research [8,9] have been studied and applied to the power quality analysis. These harmonic detection methods have their own characteristics, but all the typical analytic methods are based on the Nyquist/Shannon sampling theory. Their analytic methods are similar as they all begin with the collection of power quality data through high frequency acquisition and A/D conversion, and then this is followed by compressing the data. Although data compression can reduce the burden of transmitting mass sample data to a certain extent, a lot of hardware resources and storage space have been spent in the high-speed sampling stage before data compression. Moreover, with the development of smart grid, high frequency switch power electronic device makes harmonic components more complex and makes sampling frequencies higher, which requires more hardware resources and storage space during the Nyquist sampling process.

As a new signal processing theory, compressed sensing (CS) combines the compression process and the sampling process. The completed data compression in the sampling process can effectively reduce the utilization of hardware resources and storage space. CS has been considered for many applications including image processing [10], signal transmission [11], communication [12], etc. Although the CS theory has been applied in image processing and other fields, less research has focused on the power systems. As a promising approach, CS was discussed in connection with the application in sampling and transmitting information from large number of sensors in the smart grid communication networks, but there is no specific implementation process of the compressed sensing application [13]. In another Ref. [14], Compressive Sampling theory was employed to analyze the synchrophasor data communication in WAMS. From the description and its references, we find that "compressive sampling theory" in [14] was actually the same as "compressed sensing theory". In the literature [15], the popular CS theory was used to classify the fault area and reduce the WAN communication traffic. Another data transmission method based on compressive sensing technology was proposed in [16]. The signals were transformed into wavelet domain coefficients by means of wavelet multi-resolution analysis. Then these coefficients were sampled and wavelet inverse transforms were applied to reconstruct the original signals. Different from that in the references [15,16], this paper applies the CS theory to harmonic and inter-harmonic signal detection and fully considers the signal characteristics in electric power systems.

This paper proposes a harmonic analysis scheme based on CS technology, which can implement the function of sampling compression, signal reconstruction and harmonic detection simultaneously. Specifically, the paper mainly provides the following contributions:

(1) *Sparseness property of electrical power system harmonic signal:* The sparseness of signal is the prerequisite of using CS theory. We not only give the sparseness property of harmonic signal in electrical power system and proportion of sparseness in fundamental and harmonic components but also prove them.
(2) *Harmonics analysis scheme based on CS theory:* The scheme includes sampling process and recovery process. In the sampling process, the scheme uses the binary sparse random measurement matrix and gives the corresponding switching hardware circuit. The binary sparse measurement can effectively save the sampling storage space and reduce sampling complexity. In the recovery process, the scheme implements the function of signal reconstruction and harmonic detection simultaneously.
(3) *Spectral Projected Gradient with Fundamental Filter* (*SPG-FF*) *algorithm*: Based on the property of sparseness proportion in fundamental and harmonic components, considering the significant differences of harmonic signal's energy in the recovery process, a novel reconstruction algorithm called the SPG-FF is proposed, which can reduce the sparsity of the signal and enhance the detection and reconstruction precision of harmonic components.

The article is structured as follows. We show how CS method can be applied to harmonic analysis. Specifically, Section 2 introduces the CS theory. The proposed harmonic analysis scheme is detailed in Section 3, followed by Section 4 describing the SPG-FF algorithm of the proposed scheme. Section 5 evaluates the proposed scheme through experiments and simulation, and Section 6 concludes the paper.

## 2. Compressed sensing theory

The theory of CS [17] shows that if a signal is sparse on some basis, it can be reconstructed from a small number of measurements. Even through a small number of measured values are used to solve an optimization problem, the original signal can still be reconstructed with high probability of accuracy.

To simplify the statement, the symbol $x$ is used to define an $N$-dimensional non-sparse signal. If $x$ can be represented as $x = \mathbf{\Psi} s$ under sparse matrix $\mathbf{\Psi}$, $x$ is considered sparse. Clearly, $x$ and $s$ are equivalent representations of the same signal in different spaces or domains, with $x$ in the time domain and $s$ in the $\mathbf{\Psi}$ domain. The $M$-dimensional observation vector $y$ can be represented as $y = \mathbf{\Phi} x$, where $\mathbf{\Phi}$ is an $M \times N$ measurement matrix, with $M < N$. Finally, the original signal $x$ can be recovered accurately by the CS reconstruction algorithm. The model of CS theory is represented as

$$y = \mathbf{\Phi} x = \mathbf{\Phi}\mathbf{\Psi} s = \mathbf{\Theta} s \quad (1)$$

## 3. Harmonic characteristics analysis

### 3.1. Sparseness of harmonic signal in electrical power system

The prerequisite of using CS theory is that the original signal is sparse (named $K$-sparse). If the original signal $x$ can be represented as $x = \mathbf{\Psi} s$ under the sparse matrix $\mathbf{\Psi}$, then values of most elements of coefficient vector $s$ are less than a small positive $\varepsilon$, it is arguable that the original signal is a $K$-sparse signal.

The mathematical equation of the harmonic signal is [18]:

$$f(t) = \sum_{h=0}^{H} A_h \cos(2\pi f_h t + \varphi_h) \quad (2)$$

Here $\{A_0, f_0, \varphi_0\}$ represents fundamental component parameters, while $\{A_h, f_h, \varphi_h\}$ $(h \geqslant 1)$ represents the parameters of the harmonic component parameters, which includes $H$th harmonic components. The signal has the following properties:

**Property 1** (*Sparseness of harmonic signal in electrical power system*). With the Discrete Fourier Transform (DFT), the harmonic signal in electrical power system satisfies the sparseness requirements of compressed sensing

The proof is shown in Appendix A.

**Property 2** (*Proportion of sparseness in fundamental and harmonic components*). In the power system, fundamental components occupy a high proportion of signal sparseness, while harmonic components occupy very low proportions.