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Effects of couple stresses on the heat transfer and entropy generation rates for a flow between parallel plates with constant heat flux

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ABSTRACT

In this study, the flow and heat transfer of a fluid with couple stresses is investigated. The flow caused by the pressure gradient between parallel plates is considered as incompressible, steady and fully developed while the bottom plate is adiabatic and the upper plate is exposed to a constant heat flux. Governing equations, i.e. momentum and energy, are derived and solved analytically. Using the analytical results, effects of couple stresses on the Nusselt number as a heat transfer performance parameter and entropy generation rates in the channel are presented via graphs and tables. In addition, second law analysis is performed by calculating mean entropy generation rates and Bejan number along the channel height.

1. Introduction

Newtonian theory considers fluids as a continuous material. Thus, the size of the fluid particles and microstructure of the fluid affecting the flow properties is neglected. However, results of many real life flow problems, such as blood flow, polymer melt extrusions, colloidal suspensions applications and some type of flows of industrial fluids or lubricants etc., indicate that a theory considering the microstructure of the continuum is needed. To overcome this situation, Stokes [1] suggested couple stresses, i.e. antisymmetric stresses, and body couples in the nonsymmetrical stress tensor of the constitutive equation of fluid as the classical approach has symmetric stress tensor, namely Cauchy stress tensor, due to ignoring the rotational interaction within the continua. Rotation and interaction between the suspended particles within the fluid form a spin field in which the antisymmetric stress tensor is defined. As the size of the particles suspended in the fluid is becoming non-negligible, couple stresses take more important role in the flow field. Briefly, couple stress is the result of force and momentum transfer between suspended particles in a fluid. In addition, moment transfer between the particles of the fluid is ignored in the classical theory thus Cauchy stress tensor describing the state of stress at a point has six components whereas the nonsymmetrical stress tensor has eighteen including body moments. For instance, the couple stress theory can be used to understand the behavior of rotating particles in a fluid at rest. In this phenomenon, the particles have only angular momentum however the classical theory indicates that angular momentum is only exist as long as linear momentum takes place in the continuum then the classical approach is inadequate.

So far, a brief introduction about the concept is presented. For details of the theory of couple stresses, the reader may refer to the book by Stokes [2]. The concept attracted many researchers in recent years. Srinivasacharya and Srikanth [3] mathematically modeled the blood as a couple stress fluid in stenosed arteries since the red blood cells are regarded as suspended particles in the media. Rathod and Tanveer [4] investigated the blood as a couple stress fluid in a periodically accelerated porous medium exposed to magnetic field. Devak*ar* et al. [5] used slip boundary conditions in generalized Coutte flows of a couple stress fluid between parallel plates. They showed that the fluid behaves more likely Newtonian fluid and gains speed as the effects of couple stresses vanish. Farooq et al. [6] discussed the influence of the couple stresses on the Poiseuille flow with heat transfer between inclined parallel plates. They assumed that the couple stress parameter was constant as the Newtonian viscosity had dependence on the temperature with Reynold's law. A set of partial differential equations arising from MHD flow of a couple stress fluid were solved analytically with the help of wave parameter by Khan et al. [7]. They discussed the simplicity and feasibility of the method on the flow of couple stress fluids. Another application of a couple stress fluid in porous media

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is performed by Makinde and Eegunjobi [8]. They studied irreversibilities in a vertical channel filled with a couple stress fluid heated from walls. Malashetty et al. [9] discussed the stability of a couple stress fluid flow with Sorret effect. They showed that the Sorret effect is quite significant for the stability. Lin considered [10] couple stress fluid as a lubricant and presented a numerical study on a squeeze film bearing. The results show that the couple stress effect enhances load carrying capacity of the bearing due to the increasing fluid film pressure. Since the researches indicate that fluid with blended additives improve the performance of bearings, couple stress fluid model is applied to many bearing lubrication problems given in reference [10]. Quite recently, several approximate analytical methods, such as perturbation method and Adomian decomposition method, were applied to the couple stress fluid problems with heat transfer considering the second law analysis [11–13], and furthermore, Srinivas and Murthy studied two immiscible couple stress fluids flowing between permeable beds modeled by Darcy porous media law [14].

Many comprehensive works on basic heat transfer analysis of thermal flow problems of both Newtonian and non-Newtonian fluids are conducted during the last decade, because of the importance of thermally efficient systems leading to economical operation costs. Just to list a few, Babaie et al. [15] studied the effect of heat transfer on the flow of a power-law fluid driven by an electroosmotic pump in a slit microchannel. Dependency of viscous heating on the heat transfer of a fluid occupied between the parallel plates subjected to different heat fluxes is examined exhaustively by Sheela-Francisca and Tso [16]. Later, a similar work is expanded to a slip flow dependent of Knudsen number by Kushawaha and Sahu [17]. A study of viscous dissipation effect on a flow in a porous medium is presented by Hung and Tso [18]. They considered the flow as steady and fully developed while the walls are being heated or cooled for different cases. An extended Nusselt number to shear thickening and shear thinning fluids is obtained by Tso et al. [19]. Tunc and Bayazitoglu [20] transform the energy equation with viscous dissipation arising from steady and fully developed flows through microtubes by an eigenvalue approach and solved analytically. Bayazitoglu [21] et al. modeled a gel as Bingham fluid possessing a yield point with a linear shear stress and shear rate relation. They also presented a real life illustrative problem to discuss the reliability of their study. Aydın and Avcı [22] conducted an analytical research on a Newtonian flow with heat transfer in a microchannel. They regarded the flow as incompressible, viscous, fully developed and steady as the considered energy equation involves viscous dissipation effects.

There are many flow problems subjected to heat transfer process thus second law analysis is of importance in the design of thermally efficient systems [23]. Second law analysis deals with the entropy generation rates as a measure of irreversibilities in the system. Irreversibilities, that must be minimized in the thermodynamically efficient systems by second law analysis, arise from viscous effects besides the heat transfer. Accordingly, the viscous dissipation terms are taken into account in the energy equation to fully understand the irreversibilities of a system. Shojaeian and Koşar [24] examined the irreversibilities in a flow of a non-Newtonian fluid with slip effect on isoflux and isothermal boundaries of parallel plates. They showed that the mean entropy generation rate decreases with the increasing slip on the boundaries in contrast with increasing non-Newtonian features. Elazhary and Soliman [25] considered the entropy production in a two dimensional flow induced by electric double layer in a microchannel exposed to a uniform heat flux at walls. They discussed the influence of viscous dissipation, channel geometry and pumping power on the entropy production. Newtonian and non-Newtonian fluid flows with heat transfer through different cross sections were studied in terms of second law of thermodynamics by Mahmud and Fraser [26]. They obtained the analytical expressions of irreversibilities for both cases. Hooman [27] applied both slip and jump effects of velocity and temperature, respectively, on the walls of microscale geometries and investigated the change of entropy generation rate under these circumstances. Pakdemirli and Yılbaş [28] derived the analytical expression of entropy generation with viscous dissipation in a pipe in which a non-Newtonian flow were taking place. Additionally, a large number of numerical, analytical or experimental studies dealing with second law analysis of Newtonian or non-Newtonian fluids under different conditions have been performed in the literature of last decade, for a few examples please refer to references [29–38].

In this present work, the couple stresses on the heat transfer and entropy generation rates are investigated by the second law analysis. To the best of author's knowledge, there is not quite a lot of studies on the heat transfer analysis of couple stress fluids, although extensive work has done in Newtonian or non-Newtonian fluid mechanics area where second law analysis were also incorporated. Therefore, both mechanics of couple stress fluid and heat transfer with irreversibilities are particularly focused in the light of previous works including primarily the Stoke's theory [2].

The organization of the rest of the paper is as follows: In the second chapter, the physical configuration of problem is simply introduced and then the governing equations of the problem are derived and solved analytically in a nondimensional form. Afterward, the heat transfer aspects of the problem is examined using the temperature and velocity distributions obtained in this chapter. In the third chapter, with the aid of the second law analysis, irreversibilities in the problem is emphasized. Finally, in the last chapter, various remarks and conclusions drawn from the formulas, graphs and tables are given.

2. Governing equations

The stress tensor related to a couple stress fluid is given as [2].

$$\tau_{ij} = -p\delta_{ij} + \mu(v_{j,i} + v_{i,j}) + \eta(v_{i,jrr} - v_{j,irr}) - \frac{1}{2}\varepsilon_{ijr}(m_{ij,r} + \rho l_r) \tag{1}$$

where p is the pressure, μ is the viscosity, η is the couple stress parameter in dimension of momentum, v_i is the velocity vector, l_i is the body moment per unit mass and m is the trace of the couple stress tensor that is of the form

$$m_{ij} = 4\eta k_{ij} + 4\eta' k_{ji}. \tag{2}$$

In the above expression, η' is a material constant, k_{ji} is the vorticity gradient due to rotation vector ω_i. Since the rate of the strain tensor and the vorticity tensor are independent of each other as well as the rate of entropy production per unit mass is always greater than zero, i.e. $\dot{\gamma} \geq 0$, the Clausius-Duhem inequality holds

$$\mu \geq 0, \quad \eta \geq 0 \text{ and } \eta \geq \left|\eta'\right|. \tag{3}$$

Stokes discussed the determination of the couple stress parameter η experimentally in [1]. In the absence of body forces, for incompressible fluids, continuity equation and momentum equation in indicial notation take the following forms

$$v_{r,r} = 0 \tag{4}$$