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Collocation methods for uncertain heat convection-diffusion problem with interval input parameters

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ABSTRACT

This paper proposes a full grid interval collocation method (FGICM) and a sparse grid interval collocation method (SGICM) to solve the uncertain heat convection-diffusion problem with interval input parameters in material properties, applied loads and boundary conditions. The Legendre polynomial series is adopted to approximate the functional dependency of temperature response with respect to the interval parameters. In the process of calculating the expansion coefficients, FGICM evaluates the deterministic solutions directly on the full tensor product grids, while the Smolyak sparse grids are reconstructed in SGICM to avoid the curse of dimensionality. The eventual lower and upper bounds of temperature responses are easily predicted based on the continuously-differentiable property of the approximate function. Comparing results with traditional Monte Carlo simulations and perturbation method, the numerical example evidences the remarkable accuracy and effectiveness of the proposed methods for interval temperature field prediction in engineering.

1. Introduction

Due to the aggressive environment factors, inevitable measurement errors and incomplete knowledge, various uncertainties representing the system variability are ubiquitous in many practical engineering problems. The uncertainty analysis, which can effectively assess the impact of input uncertainties on the system output responses, has become an indispensable component in the scientific fields [1–3]. The main approaches to quantify the system uncertainties can be grouped into three categories: probabilistic method, fuzzy theory and interval analysis [4]. The probabilistic methods, such as Monte Carlo simulation, spectral analysis method and stochastic collocation method, can be considered as the most effective techniques in the probabilistic framework, where uncertainties are usually treated as random variables with probability density functions [5–7]. The fuzzy set theory, introduced by Zadeh [8], is another efficient category to model the system uncertainty, whose membership functions are decided by the subjective opinions. In the two kinds of uncertainty methodologies, a great amount of information is required to construct the precise probability density functions and membership functions of uncertain input parameters. Unfortunately, for many complex practical problems, the abundant objective information may not be easily available in the early stage of numerical analysis.

In order to overcome the shortcomings, there has been a growing interest in interval analysis theory in recent years. It is perfectly appropriate to deal with the uncertainties whose bounds are well-defined but sufficient information is missing [9]. The interval approaches aim to compute the changing ranges of output responses with respect to the interval input parameters. Given a large number of samples, the Monte Carlo simulation is still considered as the simplest method for interval analysis [10]. But due to the huge computational cost, it is commonly introduced as a referenced approach, rarely used in the practical engineering. The standard Gaussian elimination schemes provide a hypercube approximation for the united solution set of interval equations [11], but this approximation is extremely conservative because of a large number of elimination operations [12]. If the functional monotonicity is satisfied, the exact response ranges can be achieved by the vertex method using all possible combinations of the interval parameters [13], but the optima which are not on the vertex of the input space cannot be identified efficiently. The interval parameter perturbation method, proposed by Qiu et al. [14], is another widely used approach according to the small computational cost. Because

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it adopts the first-order Taylor series to approximate the functional relationship, it is not suitable to the problem with large uncertainty level and nonlinearity degree [15].

Traditional numerical prediction of the temperature field with given deterministic parameters has been attracted extensive attention. But considering the unavoidable uncertainties in practical engineering, the nondeterministic methods are more feasible [16]. Based on the generalized polynomial chaos, Xiu and Karniadakis [17] presented a random spectral decomposition method for the solution of transient heat conduction subjected to random inputs. Combining the fuzzy theory with finite difference method, a new technology named as fuzzy difference method is proposed to predict the membership functions of temperature responses [18]. For the uncertain heat transfer problem without sufficient information, Xue and Yang [19,20] estimated the temperature intervals by using the finite element method and expansion techniques. Wang et al. [21,22] developed the interval and subinterval perturbation methods to solve the heat convection-diffusion problem with uncertain-but-bounded parameters. Two inherent disadvantages in the perturbation methods need to be pointed out: one is the low computational accuracy caused by the first-order Taylor series, especially for the strongly nonlinear problem; the other is the huge computational cost caused by the large number of subinterval combinations. Comparatively speaking, the collocation methods represent great superiority in computational accuracy and effectiveness by using the high-order polynomial series and a set of deterministic computing procedure [23]. According to the easy implement, several collocation schemes have been developed to the engineering problems [24,25]. Schieche and Lang [26] made some excellent research work on the stochastic collocation method for uncertainty analysis of the thermo-convective Poiseuille flow with random parameters. In the existing stochastic collocation methods, the probabilistic moments such as expectation and variance are the most interested benefits, and they are usually calculated by using the orthogonal relationship of polynomial bases. However, it should be noted that the current research on collocation methods is mainly concentrated in the stochastic field, while the combination of collocation method and interval analysis theory is promising but mostly unexplored [27].

The purposes of the present study include: (1) developing the collocation methods combined with interval uncertainties in the mathematical theory; (2) using the interval collocation methods to deal with the heat convection-diffusion problem with interval input parameters in the engineering application. The paper is structured as follows. The basic theory of polynomial approximation for interval heat convection-diffusion problem is firstly introduced in Section 2. Subsequently, two interval collocation methods are presented in the next two sections. The first one is FGICM where the collocation points are directly constructed by the full tensor product grids. To improve the computational cost for high-dimensional problems, the Smolyak algorithm is adopted in SGICM to reconstruct the sparse grids. In Section 5, a numerical example is provided to verify the effectiveness and accuracy of the proposed methods, and we conclude the paper with a brief discussion at last.

2. Polynomial approximation

Consider a steady-state heat convection-diffusion problem with a heat source

$$\rho c u \frac{\partial T(x)}{\partial x} = k \frac{\partial^2 T(x)}{\partial x^2} + Q(x) \quad x \in \Omega \tag{1}$$

where Ω is a bounded domain; $T(x)$ stands for the temperature response; ρ, c, k denote the density, specific heat capacity and thermal conductivity, respectively; u represents the flow velocity, and $Q(x)$ is the intensity of heat source.

For the interior domain Ω bounded by Γ as shown in Fig. 1, the Dirichlet boundary condition on Γ_1 and the Neumann boundary condition on Γ_2 are given as follows

$$\begin{aligned} T &= T_s \quad \text{on } \Gamma_1 \\ -k\frac{\partial T}{\partial \mathbf{n}}\bigg| &= q_s \quad \text{on } \Gamma_2 \end{aligned} \tag{2}$$

where T_s stands for the boundary temperature; $\mathbf{n}$ denotes the normal vector of the boundary, and q_s is the boundary heat flux.

For the practical engineering heat transfer problem, due to the system complexities and insufficient information, uncertainties in material properties, applied loads and boundary conditions are unavoidable. In this paper, the uncertain input parameters whose lower and upper bounds can be determined by the limited information are defined as interval variables as follows

$$\begin{aligned} \boldsymbol{\alpha}^I &= \left(\alpha_i^I\right)_n = \left(\left[\underline{\alpha_i}, \overline{\alpha_i}\right]\right)_n = \left(\alpha_i^c + \Delta\alpha_i^I\right)_n = \left(\alpha_i^c + \Delta\alpha_i\delta_i^I\right)_n \\ &= \boldsymbol{\alpha}^{\mathbf{c}} + \Delta\boldsymbol{\alpha}\delta^I \end{aligned} \tag{3}$$

where α_i^I stands for an interval number; $\underline{\alpha_i}$ and $\overline{\alpha_i}$ are the lower and upper bounds; $\alpha_i^c = (\overline{\alpha_i} + \underline{\alpha_i})/2$ and $\Delta\alpha_i = (\overline{\alpha_i} - \underline{\alpha_i})/2$ are called the midpoint and the radius, respectively; δ_i^I denotes the standard interval $\delta_i^I = [-1, 1]$, and n is the number of interval input parameters.

Consequently, the temperature response $T(x)$ become uncertain with respect to the interval parameter vector $\boldsymbol{\alpha}^I$. Therefore, the governing Eq. (1) with interval input parameters can be rewritten as

$$\begin{aligned} &\rho\left(\boldsymbol{\alpha}^I\right)c\left(\boldsymbol{\alpha}^I\right)u\left(\boldsymbol{\alpha}^I\right)\frac{\partial T\left(x, \boldsymbol{\alpha}^I\right)}{\partial x} \\ &= k\left(\boldsymbol{\alpha}^I\right)\frac{\partial^2 T\left(x, \boldsymbol{\alpha}^I\right)}{\partial x^2} + Q\left(x, \boldsymbol{\alpha}^I\right) \quad \left(x, \boldsymbol{\alpha}^I\right) \in \Omega \times \mathrm{V} \end{aligned} \tag{4}$$

where V denotes the interval uncertain space.

As we know, the traditional interval analysis methods based on Taylor series only use the midpoint's information to approximately calculate maximum and minimum values of the system responses, whose accuracy for the nonlinear function becomes unacceptable if the fluctuation level of the expansion parameters is not small enough. In order to overcome the shortcoming, we will adopt the high-order polynomial series to approximate the temperature response in this paper. Based on the interval central expression as

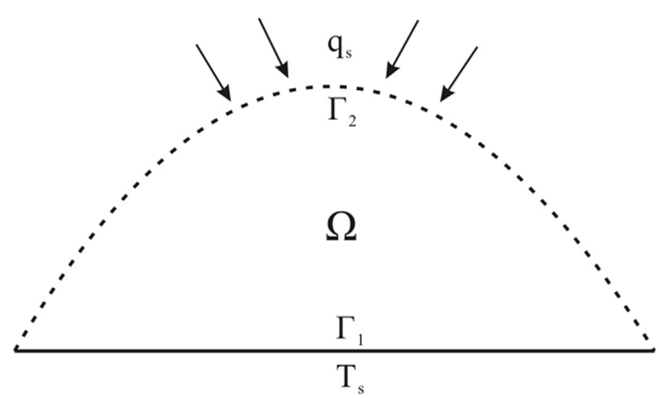

Fig. 1. Boundary conditions of heat convection-diffusion problem.