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Natural convective boundary-layer flow of a nanofluid past a vertical plate: A revised model

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ABSTRACT

The problem of natural convective boundary-layer flow of a nanofluid past a vertical plate is revisited. The model, which includes the effects of Brownian motion and thermophoresis, is revised so that the nanofluid particle fraction on the boundary is passively rather than actively controlled. In this respect the model is more realistic physically than that employed by previous authors.

1. Introduction

The model for a nanaofluid including the effects of Brownian motion and thermophoresis, introduced by Buongiorno [1], was applied by Kuznetsov and Nield [2] to the classical problem studied by Pohlhausen, Kuiken and Bejan [3–6], namely convective boundary layer flow past a vertical plate. In their pioneering paper Kuznetsov and Nield [2] employed boundary conditions on the nanoparticle fraction analogous to those on the temperature. In this note the problem is revisited and a boundary condition that is more realistic physically is applied. It is no longer assumed that one can control the value of the nanoparticle fraction at the wall, but rather that the nanoparticle flux at the wall is zero. This change necessitates a rescaling of the parameters that are involved.

2. Analysis

The following analysis closely follows that in Ref. [2] and so is described briefly here. A two-dimensional problem is considered. A coordinate frame in which the x-axis is aligned vertically upwards is utilized. A vertical plate is at $y = 0$. We assume that at $y = 0$ the temperature T takes the constant value T_w. The flux of the nanoparticle fraction at $y = 0$ is taken to be zero. The ambient value of temperature is T_∞ and the ambient value of the nanoparticle fraction is ϕ_∞; the ambient values are attained at an infinite distance from the wall.

We used the Oberbeck-Boussinesq approximation. The governing equations expressing the conservation of total mass, momentum, thermal energy, and nanoparticles are, respectively:

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

$$\rho_f\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \mu\nabla^2\mathbf{v} + \Big[\phi\rho_p + (1-\phi) \times \Big\{\rho_f(1-\beta(T-T_\infty))\Big\}\Big]\mathbf{g}, \tag{2}$$

$$(\rho c)_f\left(\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T\right) = k\nabla^2 T + (\rho c)_p\left[D_B\nabla\phi\cdot\nabla T + (D_T/T_\infty)\nabla T\cdot\nabla T\right], \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \mathbf{v}\cdot\nabla\varphi = D_B\nabla^2\phi + (D_T/T_\infty)\nabla^2 T. \tag{4}$$

In Eqs. (1)–(4) the field variables are the velocity $\mathbf{v}$ [we write $\mathbf{v} = (u,v)$], the temperature T and the nanoparticle volume fraction

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Nomenclature

D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
f	rescaled nanoparticle volume fraction, defined by Eq. (20)
$\mathbf{g}$	gravitational acceleration vector
k	thermal conductivity
Le	Lewis number, defined by Eq. (28)
Nr	buoyancy–ratio parameter, defined by Eq. (25)
Nb	Brownian motion parameter, defined by Eq. (26)
Nt	thermophoresis parameter, defined by Eq. (27)
Nu	Nusselt number, defined by Eq. (31)
Nur	reduced Nusselt number, $Nu/Ra_x^{1/4}$
Pr	Prandtl number, defined by Eq. (24)
p	pressure
q''	wall heat flux
Ra_x	local Rayleigh number, defined by Eq. (18)
s	dimensionless stream function, defined by Eq. (20)
T	temperature
T_w	temperature at the vertical plate
T_∞	ambient temperature attained as y tends to infinity
$\mathbf{v}$	velocity, (u, v)
(x, y)	Cartesian coordinates (x-axis is aligned vertically upwards, plate is at $y = 0$)

Greek symbols

α	thermal diffusivity
β	volumetric expansion coefficient of the fluid
η	similarity variable, defined by Eq. (19)
θ	dimensionless temperature, defined by Eq. (20)
μ	dynamic viscosity of the fluid
ν	kinematic viscosity, $\mu/\rho_{f\infty}$
ρ_f	fluid density
ρ_p	nanoparticle mass density
$(\rho c)_f$	heat capacity of the fluid
$(\rho c)_p$	effective heat capacity of the nanoparticle material
τ	parameter defined by Eq. (13), $(\rho c)_p/(\rho c)_f$
ϕ	nanoparticle volume fraction
ϕ_∞	ambient nanoparticle volume fraction attained as y tends to infinity
ψ	stream function, defined by Eq. (14)

ϕ. Also, ρ_f is the density of the base fluid and μ, k and β are the viscosity, thermal conductivity and volumetric expansion coefficient of the nanofluid, and ρ_P is the density of the particles. We denoted the gravitational acceleration by $\mathbf{g}$. In Eqs. (3) and (4) the coefficients D_B and D_T are the Brownian diffusion coefficient and the thermophoretic diffusion coefficient, respectively, each nondimensionalized in terms of the ambient value of the temperature. It is being assumed the temperature does not vary much from the ambient temperature, and so D_B and D_T may each be treated as a constant.

Eqs. (1)–(4) must be solved subject to the following boundary conditions:

$$u = v = 0,\ T = T_w,\ D_B\frac{\partial\phi}{\partial y} + \frac{D_B}{T_\infty}\frac{\partial T}{\partial y} = 0 \text{ at } y = 0, \tag{5a,b,c}$$

$$u = v = 0,\ T \to T_\infty,\ \phi \to \phi_\infty \text{ as } y \to \infty. \tag{6a,b,c}$$

A steady state flow is considered. Eq. (5c) is a statement that, with thermophoresis taken into account, the normal flux of nanoparticles is zero at the boundary [7,8].

We used the Oberbeck-Boussinesq approximation. We also made an assumption that the nanoparticle concentration is dilute. Using a suitable choice for the reference pressure, we linearized the momentum equation and recast Eq. (2) as follows:

$$\rho_f\left(\frac{\partial\mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla p + \mu\nabla^2\mathbf{v} + \Big[\left(\rho_p - \rho_{f\infty}\right)(\phi - \phi_\infty) + (1 - \phi_\infty)\rho_{f\infty}\beta(T - T_\infty)\Big]\mathbf{g}. \tag{7}$$

Based on a scale analysis, we now employ the standard boundary-layer approximation, and express the governing equations as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{8}$$

$$\frac{\partial p}{\partial x} = \mu\frac{\partial^2 u}{\partial y^2} - \rho_f\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + \Big[(1 - \iota_\infty)\rho_{f\infty}\beta g(T - T_\infty) - \left(\rho_p - \rho_{f\infty}\right)g(\phi - \phi_\infty)\Big], \tag{9}$$

$$\frac{\partial p}{\partial y} = 0, \tag{10}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\nabla^2 T + \tau\left[D_B\frac{\partial\phi}{\partial y}\frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{11}$$

$$u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y} = D_B\frac{\partial^2\phi}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2}, \tag{12}$$

where

$$\alpha = \frac{k}{(\rho c)_f},\ \tau = \frac{(\rho c)_p}{(\rho c)_f}. \tag{13}$$

We used cross-differentiation to eliminate p from Eqs. (8) and (9). We also introduced a stream function ψ defined by

$$u = \frac{\partial\psi}{\partial y},\ v = -\frac{\partial\psi}{\partial x}. \tag{14}$$

Eq. (8) is now satisfied identically.

This leaves us with the following three equations:

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} - \nu\frac{\partial^3\psi}{\partial y^3} = (1 - \phi_\infty)\rho_{f\infty}\beta g(T - T_\infty) - \left(\rho_p - \rho_{f\infty}\right)g\phi \tag{15}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y} = \alpha\nabla^2 T + \tau\left[D_B\frac{\partial\phi}{\partial y}\frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{16}$$