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Local thermal non-equilibrium effects on the onset of convection in an internally heated layered porous medium with vertical throughflow

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ABSTRACT

We investigated how local thermal non-equilibrium and vertical throughflow affect the stability of an internally heated fluid-saturated porous layer. In order to investigate the effects of heterogeneity, we considered a system composed of two horizontal porous layers with different properties. This allowed us to investigate the effects of vertical variation of various parameters. Due to a large number of parameters, our primary goal was to investigate which parameters have a significant effect on the stability of the system. We have found that heterogeneity of permeability and fluid thermal conductivity have a major effect, heterogeneity of interphase heat transfer coefficient and porosity have a lesser effect, while heterogeneity of solid thermal conductivity is relatively unimportant. At the same time we investigated the variation of both upward and downward throughflow, and variation of heat source strength between the layers and between the fluid and solid phases. Downward throughflow is destabilizing, while upward throughflow is stabilizing. The stability is strongly affected by the solid-to-fluid heat source strength ratio.

1. Introduction

In the standard formulation of the onset of convection in a horizontal layer of a porous medium with symmetrical (top/bottom) perturbation boundary conditions (for example, both boundaries conducting and impermeable) the solution for the convective flow is symmetrical about the horizontal mid-plane of the layer. This symmetry can be removed in various ways, such as by vertical heterogeneity or by vertical throughflow, or if the layer is internally heated, and separately each of these situations has been comprehensively studied (see, for example, the surveys in Sections 6.13, 6.10.2 and 6.5 of Nield and Bejan [1]). In the case of vertical throughflow the asymmetry is a result of the basic vertical temperature gradient being augmented in the region close to the boundary towards which the throughflow is directed, with a corresponding reduction in that gradient in the rest of the layer. In the case of internal heating a negative basic temperature gradient is present only in the upper portion of the system and so the convection, when it occurs, is concentrated in that portion. That means that relatively greater permeability or relatively smaller fluid thermal conductivity in the upper region each reduce the stability. Local thermal non-equilibrium (LTNE) provides a further complication. Our present interest lies in the effects of the combination of heterogeneity, throughflow and LTNE. The effect of throughflow with LTNE has been investigated by Patil and Rees [2], but only in the case where the throughflow is large enough to produce a thermal boundary layer at the boundary towards which the flow is directed. The combined effects of heterogeneity (in the special form of two layers each of which is homogeneous) and vertical throughflow have been studied by Nield and Kuznetsov [3]. However, it appears that, to the best of our knowledge, the combination of LTNE and vertical throughflow had not been investigated in any previous study. This situation is of interest because the throughflow occurs in the fluid phase only, and this has ramifications both for the basic solution and the instability problem. We also mention that Nield and Kuznetsov [4] studied the effects of LTNE in a layered porous medium for the case of bottom heating, while the case of LTE for internal heating in a layered porous medium was treated by Nield and Kuznetsov [5].

For the case of a layered medium with internal heating, studies have been made by Kuznetsov and Nield [6–8]. In Ref. [7] throughflow was considered. In Ref. [8] LTNE was treated.

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Nomenclature

a	dimensionless horizontal wavenumber
D	d/dz
$\mathbf{e}_z$	unit vector in the z-direction
h	interface heat transfer coefficient (incorporating the specific surface area) between the fluid and solid particles
$\widehat{h}$	parameter defined in Eq. (15)
h_r	interface heat transfer coefficient ratio, h_2/h_1
g	gravitational acceleration
$\mathbf{g}$	gravitational acceleration vector
H	dimensional layer depth
k	thermal conductivity of the porous medium
$\widehat{k}_f$	parameter defined in Eq. (15)
k_{fr}	fluid thermal conductivity ratio, k_{f2}/k_{f1}
$\widehat{k}_s$	parameter defined in Eq. (15)
k_{sr}	solid thermal conductivity ratio, k_{s2}/k_{s1}
K	permeability of the porous medium
$\widehat{K}$	parameter defined in Eq. (15)
K_r	permeability ratio, K_2/K_1
N	interface heat transfer parameter, $h_1H^2/(\phi_1 k_{f1})$
P	dimensionless pressure, $((\rho c)_f K_1/(\mu k_{f1}))P^*$
P^*	pressure, excess over hydrostatic
Pe	Péclet number, $(\rho c)_f HV/k_{f1}$
Q_f	volumetric heat source strength in the fluid phase
$\widehat{Q}_f$	parameter defined in Eq. (15)
Q_{fr}	volumetric heat source strength ratio in the fluid phase, Q_{f2}/Q_{f1}
Q_s	volumetric heat source strength in the solid phase
$\widehat{Q}_s$	parameter defined in Eq. (15)
Q_{sr}	volumetric heat source strength ratio in the solid phase, Q_{s2}/Q_{s1}
Ra	internal Rayleigh number, $(\rho c)_f \rho_0 g\beta K_1 H^3 Q_{f1}/(2\mu k_{f1}^2)$
t	dimensionless time, $(k_{f1}/((\rho c)_f H^2))t^*$
t^*	time
T	dimensionless temperature, $((\rho c)_f \rho_0 g\beta K_1 H/(\mu k_{f1}))(T^* - T_0)$
T^*	temperature
T_0	temperature at the lower and upper boundaries
(u,v,w)	dimensionless velocity components, $((\rho c)_f H/k_{f1})(u^*, v^*, w^*)$
$\mathbf{u}^*$	Darcy velocity, (u^*,v^*,w^*)
V	vertical throughflow velocity
(x,y,z)	dimensionless Cartesian coordinates, $(x^*, y^*, z^*)/H$
(x^*,y^*,z^*)	Cartesian coordinates; z^* is the vertically-upward coordinate

Greek symbols

α	modified thermal diffusivity ratio, $((\rho c)_{s1}/(\rho c)_{f1})(k_{f1}/k_{s1})$
β	volumetric expansion coefficient of the fluid
γ	modified thermal conductivity ratio, $\phi_1 k_{f1}/((1-\phi_1)k_{s1})$
δ	dimensionless layer depth ratio (interface position)
$\widehat{\delta}$	parameter defined in Eq. (15)
δ_r	inverse solid fraction ratio, $(1-\phi_1)/(1-\phi_2)$
ε	dimensionless small quantity
$\widehat{\varepsilon}$	parameter defined in Eq. (15)
ε_r	solid heat capacity ratio, $(\rho c)_{s2}/(\rho c)_{s1}$
μ	fluid viscosity
ρ_0	fluid density at temperature T_0
ρ_f	fluid density
ρ_Q	Q_{s1}/Q_{f1}
$(\rho c)_f$	heat capacity of the fluid
$(\rho c)_m$	effective heat capacity of the porous medium
$(\rho c)_s$	heat capacity of the solid
ϕ	porosity
$\widehat{\phi}$	parameter defined in Eq. (15)
ϕ_r	porosity ratio, ϕ_2/ϕ_1

Subscripts

B	basic state
c	critical value
f	fluid phase
m	effective property for the porous medium
r	relative quantity
s	solid phase
1	the region $0 \le z^* < \delta H$
2	the region $\delta H \le z^* \le H$

Superscripts

$'$	perturbation variable
$*$	dimensional variable

Since the effect of throughflow is the primary concern in this paper, we document the literature on that topic. Research up to mid-2012 is surveyed in Section 6.10.2 of Nield and Bejan [1]. The early studies were by Sutton [9], Homsy and Sherwood [10] and Nield [11]. In the case of large Péclet number the effect of throughflow is to confine significant thermal gradients to a thermal boundary layer at that boundary toward which the throughflow is directed. The effective vertical length scale is thus reduced, and since the effective Rayleigh number is proportional to this length scale, it immediately follows that larger values of the Rayleigh number are needed before convection begins. For smaller Péclet numbers the effect depends on whether the thermal boundary conditions are symmetrical or not. Interesting recent papers include those by Patil and Rees [2], and by Capone, De Luca and colleagues [12–15].

For completeness, in the present paper we provide a grand coalescence with all four effects: those of LTNE, layered medium, throughflow, and internal heating.

2. Analysis

Asterisks are used to denote dimensional variables. The z^*-axis is taken in the upward vertical direction, and the porous medium is unbounded in the x^* and y^* directions. Subscripts 1 and 2 are used to denote the two layers, of depths δH and $(1-\delta)H$, where δ is less than unity.

The first layer occupies the region $0 \le z^* < \delta H$ and the second layer occupies the region $\delta H < z^* \le H$. We suppose that the upward uniform vertical throughflow Darcy velocity is V, a specified quantity. Continuity of mass requires that this is the same in each region. Thus V is a constant. The fluid and solid phases are denoted by subscripts f and s. A uniform temperature T_0 is imposed at each of the lower and upper boundaries in each phase. Fluid and solid volumetric heat sources of strengths Q_{f1}, Q_{s1} and Q_{f2}, Q_{s2} occupy the lower and upper layers, respectively.