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Review

Transient heat conduction within periodic heterogeneous media: A space-time homogenization approach

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ABSTRACT

Composite materials generally exhibit a highly anisotropic thermal behavior (due to the orientation of fibers), leading to strong difficulties to determine the thermal conductivity tensor. Two approaches can be developed for its evaluation. The first one is to carry out experimental measurements with one or several devices to get all the components of the tensor. The second one is to use predictive models based on homogenization theories from the properties and the arrangement at the microscopic scale of each component of the composite material.

Following this second approach, a space-time homogenization based on the multi-scale asymptotic expansion method is first developed to model the transient heat conduction problem within periodic heterogeneous structures. The introduction of additional terms to correct the edge effects (i.e. close to the boundaries of the macroscopic domain) in the transient state is considered. We show how these transient correcting terms can be introduced and calculated, depending on the classical boundary conditions in conduction heat transfer problems. Moreover, we underline that correcting terms have also to be added to take into account “short time” effects.

Furthermore, we propose to discuss numerical results of the heat transfer modeling in a Laser Flash experiment. We specifically show how the effective thermal diffusivity may be biased when edge effects are neglected in the homogenized model.

1. Introduction

In order to predict the thermal (and the associated thermo-mechanical) behavior within complex heterogeneous media, a usual approach consists in determining the effective thermal properties and/or temperature fields through homogenized heat transfer models. This approach is very interesting in several industrial domains such as in aeronautics or automotive. A typical example is the design of complex parts for new airplane structure where the use of composite materials with highly anisotropic thermal properties (due to the orientation of fibers) becomes more and more systematic [1]. However, the determination of the effective thermal conductivity tensor is a tricky task. Consequently, reliable and efficient methods, initially developed for mechanical models [2], are thus required for its determination. From an experimental point of view, specific devices such as classical transient laser flash (LF) [3,4], hot wire [5] methods, or a specific hot disc method [6] can be used to estimate the effective thermal properties. Another possibility is to make calculations from a representative volume element [7] of the anisotropic actual medium, knowing the thermal conductivity of each phase.

The heat transfer modeling according to a multi-scale analysis [8–10] is complementary to the experimental approach and quite powerful. It aims on one hand to determine the effective thermal properties from data known at the scale of the components, and on the other hand to have a better understanding of the “edge effects” [11–14] which may disrupt the temperature field of the homogenized heat conduction model close to the boundaries of the spatial domain. To our knowledge, this second aspect is rarely discussed in the literature. Consequently, within the framework of the estimation process of effective properties, which would result on the comparison of experimental surface temperatures with the solutions of homogenized models, more insight have to be done in the analysis of these “edge effects”, to know when they can be

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neglected or not. More generally, it is well-known that errors associated to modeling in the inverse analysis of experimental data should imply biased estimation (systematical errors) of the unknown model parameters [15].

A previous work [13], devoted to the heat conduction steady state analysis within heterogeneous periodic structures, shows how correcting terms can be introduced in the multi-scale asymptotic method to take into account these "edge effects", in the case of a homogenized 3-D heat conduction problem. The results were obtained by following the works of Dumontet [14] in elasticity.

In this paper, a space-time homogenization approach, based on the multi-scale asymptotic expansion method, is developed first to model the transient heat conduction problem within periodic structures. Such approach was also studied in Refs. [16,17]. However, the introduction of additional terms to correct the edge effects in transient state was not considered. We show how these transient correcting terms can be introduced and calculated, depending on the classical boundary conditions in heat transfer problems. Moreover, correcting terms have also to be added to take into account "short time" effects. Numerical results are presented in the case of a simple multilayered media, but the method is quite general and it could be used for periodic heterogeneous structures, like in plain weave fabric composites [18].

The last section is devoted to the discussion of numerical results of the heat transfer modeling in a LF experiment where numerical data of Fudym et al. [11] are thus used for this purpose. The heterogeneous solution is compared to the homogenized one, computed with and without correcting terms, and to the analytical homogeneous solution. Specifically, the LF method is based on the heating of the front surface of a thin sample (with parallel faces) with a nearly instantaneous pulse of light (compared to the heat conduction characteristic time of the medium). The influence of the heat losses by convection is also considered. The temperature rise on the back face is measured as a function of time (thermocouple or IR detector), and it is used to determine the thermal diffusivity (in the direction normal to the back face) of the sample. The bias, due to the estimated value of the thermal diffusivity when edge effects are neglected in the homogenized model, is evaluated.

2. Problem statement heat conduction in the heterogeneous material

Let us consider a piece of heterogeneous periodic material, Fig. 1, defined in a bounded domain $\Omega \subset \mathrm{R}^3$. The macroscopic coordinates of a point of Ω are denoted $\boldsymbol{x} = (x_1, x_2, x_3)$ in a Cartesian coordinate system $\{\mathbf{0}, \boldsymbol{e_1}, \boldsymbol{e_2}, \boldsymbol{e_3}\}$. The boundary $\partial\Omega$ is subdivided in four distinct parts $\partial\Omega = \cup_{i=1}^{4} \Gamma_i$, in order to consider the different usual kinds of boundary conditions associated to the heat conduction problem:

- A Fourier condition on Γ_1: the normal outward component $\boldsymbol{\phi}^{\boldsymbol{\varepsilon}} \cdot n$ of the heat flux is fixed by an external temperature T_{ext} and a heat transfer coefficient h.
- A Neumann condition on Γ_2: the normal outward component $\boldsymbol{\phi}^{\boldsymbol{\varepsilon}} \cdot n$ of the heat flux is fixed.
- A Dirichlet condition on Γ_3: the temperature is imposed.
- A periodic condition on Γ_4.

The initial condition is defined by the field $T_{ini}(\boldsymbol{x})$, $\boldsymbol{x} \in \Omega$, which is isothermal or not. A spatially distributed volume heat source $f(\boldsymbol{x})$, $\boldsymbol{x} \in \Omega$ may be considered all over the spatial domain, at the macroscopic scale.

The heterogeneous fields in the spatial domain Ω, are denoted respectively T^ε (temperature) and ϕ^ε (heat flux density). These fields over the time interval $(0, t_f)$ satisfy the following set of transient heat conduction equations together with the different kinds of boundary and initial conditions:

$$\begin{cases} \rho C_p \dfrac{\partial T^\varepsilon(\boldsymbol{x}, t)}{\partial t} - div_x[\boldsymbol{\phi}^{\boldsymbol{\varepsilon}}(x, t)] = f(x) \quad in \quad \Omega \times \left(0, t_f\right) \\ \boldsymbol{\phi}^{\boldsymbol{\varepsilon}}(\boldsymbol{x}, t) = \boldsymbol{K} \boldsymbol{\nabla_x} T^\varepsilon(\boldsymbol{x}, t) \quad in \quad \Omega \times \left(0, t_f\right) \\ T^\varepsilon(x, t = 0) = T_{ini}(\boldsymbol{x}) \quad in \quad \Omega \\ \boldsymbol{\phi}^{\boldsymbol{\varepsilon}}(\boldsymbol{x}, t) \cdot n = h(T^\varepsilon - T_{ext}) \quad on \quad \Gamma_1 \times \left(0, t_f\right) \\ \boldsymbol{\phi}^{\boldsymbol{\varepsilon}}(\boldsymbol{x}, t) \cdot \boldsymbol{n} = F \quad on \quad \Gamma_2 \times \left(0, t_f\right) \\ T^\varepsilon(x, t) = T_{ext} \quad on \quad \Gamma_3 \times \left(0, t_f\right) \\ Periodic\ conditions\ on\ \Gamma_4 \times \left(0, t_f\right) \end{cases} \tag{1}$$

where $\boldsymbol{n}$ is the outward normal unit; ρ, C_p and $\boldsymbol{K}$ are respectively the density, the heat capacity and the thermal conductivity tensor of the heterogeneous medium which is assumed to have a periodic structure.

The periodic cell (see Fig. 1), is denoted $Y = \prod_{i=1}^{3}[0, l_i]$ and $\boldsymbol{y} = (y_1, y_2, y_3) \in Y$ are the coordinates of a cell point. The scale factor ε is the ratio between the size of Y and the size of Ω, the microscopic coordinates are thus defined from $\boldsymbol{y} = \varepsilon^{-1}\boldsymbol{x}$.

Each component $k_{ij}(\boldsymbol{y})$; $i,j = 1, 2, 3$ of the thermal tensor and the parameter $\rho C_p(\boldsymbol{y})$ are cell-periodic and depend on the local variable $\boldsymbol{y}$ (microscopic scale) in the cell domain Y.

3. Periodic homogenization in transient state

3.1. Multi-scale asymptotic expansion method

It is assumed that the thermal conductivity of each components of the heterogeneous structure have the same order of magnitude, which means that the thermal contrast is not too large. The same assumption is done for the heat capacities. The influence of large contrast is not considered here and should lead to more developments, as described for example in Ref. [10].

Assuming that the scale factor ε is small enough, the asymptotic expansion method [9] may be used and the temperature T^ε is expanded, like for steady state solutions [13], under the following form:

$$T^\varepsilon(\boldsymbol{x}, \boldsymbol{y}, t) = T^0(\boldsymbol{x}, \boldsymbol{y}, t) + \varepsilon T^1(\boldsymbol{x}, \boldsymbol{y}, t) + \varepsilon^2 T^2(\boldsymbol{x}, \boldsymbol{y}, t) + \ldots; \quad \boldsymbol{x} \in \Omega, \boldsymbol{y} \in Y \tag{2}$$

where T^k is the approximation of T^ε at the order k and is supposed to be spatially periodic at the microscopic scale. The time variable appears in the asymptotic development as a simple parameter. Consequently, the heat flux density $\boldsymbol{\phi}^\varepsilon(\boldsymbol{x},t) = \boldsymbol{K}\boldsymbol{\nabla_x} T^\varepsilon(\boldsymbol{x},t)$ can be expanded such that:

$$\boldsymbol{\phi}^\varepsilon(\boldsymbol{x}, \boldsymbol{y}, t) = \varepsilon^{-1}\boldsymbol{\phi}^{-\mathbf{1}}(\boldsymbol{x}, \boldsymbol{y}, t) + \boldsymbol{\phi}^{\mathbf{0}}(\boldsymbol{x}, \boldsymbol{y}, t) + \varepsilon\boldsymbol{\phi}^{\mathbf{1}}(\boldsymbol{x}, \boldsymbol{y}, t) + \ldots \tag{3}$$

By injecting the development (2) in the transient heat conduction Equation (1), and using the property that an entire series is equal to zero if and only if each of its term is null [8–10], it results in a new equation for each power of ε:

- At the order $k = -2$, it comes: $div_{\boldsymbol{y}}[\boldsymbol{K}(\boldsymbol{y})\boldsymbol{\nabla_y} T^0(\boldsymbol{x}, \boldsymbol{y}, t)] = 0$. Furthermore, since T^0 is periodic, it implies that $\boldsymbol{K}(\boldsymbol{y})\boldsymbol{\nabla_y} T^0(\boldsymbol{x}, \boldsymbol{y}, t) = 0$. Hence the function T^0 does not depend on the microscopic variable: $T^0(\boldsymbol{x}, \boldsymbol{y}, t) = T^0(\boldsymbol{x}, t)$. Consequently, $\boldsymbol{\phi}^{-\mathbf{1}}(\boldsymbol{x}, \boldsymbol{y}, t) = \boldsymbol{K}(\boldsymbol{y})\boldsymbol{\nabla_y} T^0(\boldsymbol{x}, t) = 0$.