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Effect of choked outlet on transient energy growth analysis of a thermoacoustic system

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HIGHLIGHTS

- Transient energy growth analysis of a choked thermoacoustic system is conducted.
- Both analytical and numerical methods are used to describe the choked boundary.
- Two energy measures are defined and compared.
- Entropy fluctuations should be included in the transient growth analysis.
- The present work opens up new way to predict transient stability behaviors.

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ABSTRACT

Thermoacoustic instability occurs in many practical combustion systems. These systems are non-normal and associated with transient growth of acoustic disturbances. If the transient growth is large enough, then thermoacoustic instability may be triggered. In this work, transient energy growth analysis of a thermoacoustic system with a choked outlet is conducted. The effect of the choked boundary is studied by using an analytical and a linearized Euler equation (LEE) method. To quantify the transient growth, two energy measures are defined and calculated. One is associated with the acoustical energy. The other is the total energy of both entropy and acoustic fluctuations. Comparison is made between the transient growth results obtained from the analytical method and those from the LEE method. It is found that the transient growth analysis of the total energy by using the analytical model with the expression of the choked outlet is consistent with that by using the LEE method. However, when only acoustical energy is considered, the analytical model may leads to a wrong prediction of transient growth. The present work opens up new applicable way to predict transient stability behaviors of a practical engine system ended with a choked outlet.

1. Introduction

Lean premixed pre-vaporized (LPP) combustion technology is widely applied in order to reduce NOx emissions from modern gas turbines. However, the engine systems with LPP applied are more susceptible to self-sustained combustion oscillations (also known as thermoacoustic instability). It may result in engine structural damages. Intensive research has been conducted to understand the physics of thermoacoustic instability and to develop mitigation methods in the past few decades [1]. However, it is difficult to comprehensively understand the fundamental physics of such instability, because the combustion systems involve various and complex processes, such as heat transfer, fluid dynamics, chemical combustion, interaction of flame and acoustic wave, and coupling of acoustic and entropy disturbances at the boundaries [2–5].

Linear stability analysis is generally conducted as a conventional tool for studying the stability behaviors of a thermoacoustic system [6,7]. Such linear analysis is associated with eigenfrequencies prediction. A thermoacoustic system is linearly stable, when all eigenmodes decay exponentially. However recent researches show that thermoacoustic systems are generally non-normal and associated with non-orthogonal eigenmodes [8–10]. In such non-normal systems, transient energy growth of flow disturbances can occur. If the transient growth is large enough, limit cycle thermoacoustic oscillations can be triggered even for a linearly stable system [1,11]. This means that it is not enough to predict the sta-

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bility behaviors of such non-normal combustion system by simply calculating its eigenfrequencies.

It has been shown [8,9,12,13] that the transient growth of acoustic disturbances and the non-orthogonality of the eigenmodes are caused by unsteady heat release [8,9], non-trivial boundary conditions [12] and the temperature ratio across the heat source [13]. In many previous works, the mean flow effects are ignored, and the disturbances in thermoacoustic system are then investigated through Galerkin expansion method [8,9]. However, when the flow is present, the entropy disturbances can be generated and convected at the flow speed. Under this condition, the disturbance energy in the flow should include the entropy terms [14,15]. It has been proved that the entropy disturbances have significant effects on the transient growth of a thermoacoustic system. Wieczorek et al. [12] investigated the transient growth of the total energy of the thermoacoustic system based on the Myers' energy norm [15], which includes both entropy and acoustic disturbances.

The entropy disturbances can generate acoustic waves in the accelerating mean flow [16]. Therefore the effects of entropy disturbances on thermoacoustic instability become more significant in a system with choked outlet. It is necessary to know the effect of choked outlet on acoustics characteristics, such as reflection and/or transmission of pressure waves and entropy waves. Several groups of researchers have attempted to study this problem in the absence of heat sources. Candel and Marble [17] obtained an analytical solution of a choked nozzle with the assumption that the nozzle is compacted. Their expression has been widely used to predict the stability behaviors of a combustor with a choked end [7,18–23]. Then Stow et al. [24] extended the study of Marble and Candel by making a second order correction and proposing a length correction to describe the choked end. Recently Goh and Morgans [25] theoretically determined the effective length of downstream of a choked combustor. Duran and Moreau [26] used invariants method and obtained the solutions of the quasi-1D LEE. Good agreement is observed between their results and the numerical ones in terms of both modulus and phase.

It can be shown that Marble and Candel's expression can provide accurate solution as numerical model do, when the choked nozzle is acoustically compacted [7]. However if the length of the choked nozzle is not small (not acoustically compacted), this expression is not accurate any more [7]. In Ref. [24], the expression of choked outlet with a length correction is given and validated in comparison with the numerical results of linear Euler equations. It was found that when the length correction is applied, the phase of the reflection coefficient agrees well with the numerical results [24]. But the modulus is also different from the numerical predictions [24]. In physics when these expressions for choked outlet are used, there will be some spurious reflected acoustic energy, and the instability may be over predicted. The transient energy growth behavior of a thermoacoustic system may be very sensitive to the energy change in the system. Hence the expression of choked outlet may have great effects on the analysis of transient growth. But it lacks such investigations. Therefore in the present paper, like in Ref. [24], the errors brought by the boundary condition expression in calculation of transient growth will be assessed by comparing the results of analytical model and LEE method. In addition, thermoacoustic oscillations are of particular current concern due to their frequency occurrence in the new generation of gas turbines and many other engine systems, such as aeroengine afterburners, rocket motors, ramjets and boilers. Thus understanding of the transient energy growth of thermoacoustic oscillations and its prediction are important. This partially motivates the present investigation.

In this work, transient energy growth of a choked combustor is conducted based on analytical and numerical LEE method. The geometry and physical configuration of the choked combustor with an acoustically compact flame confined is described in Section 2. The definitions of two different energy measures characterizing transient growth are introduced in Section 3. The effect of the choked boundary on modal and transient growth analysis of the thermoacoustic combustor is studied in Section 4. Both the analytical and numerical methods describing the choked outlet are also described. The results obtained from the analytical and numerical methods are compared and discussed in details in Section 5. The key findings are summarized in Section 6.

2. Geometry and physical configuration of the choked combustor

An one dimensional choked thermoacoustic system is considered in the present paper. The main configuration of the choked thermoacoustic system is shown in Fig. 1(a). Its total length is L. And it consists of a cylindrical duct with a constant cross-sectional area A_p and a length L_u and a choked outlet of length $L - L_u$. An acoustically compact flame is anchored at $x = L_f$. The flow is assumed to be isentropic and uniform in the pre- and after-combustion regions. The Mach number in the pre- and after-combustion regions are denoted by M_1 and M_2 respectively. P_{01} and T_{01} denote the total pressure and total temperature in the pre-combustion region. P_{02} and T_{02} describe the downstream ones. Subscripts 1 and 2 denote the pre-and after-heat input regions respectively. A similar physical configuration is discussed in Ref. [7]. And it can be simplified to the configuration as shown in Fig. 1(b). The choked outlet is well described in terms of acoustics characteristics by using an effective length and Marble & Candle expression [17]. This is confirmed later by comparing with LEE simulation. The choked outlet is designed based on the classical isentropic mean flow equations. The flow Mach number has been shown to be spatially dependent as given as [7]

$$M(x) = M^{in} + (M^{out} - M^{in})\left(\frac{x - L_u}{L - L_u}\right)^3 \tag{1}$$

for $x > L_u$. Here, $M(x)$ denotes the Mach number in the nozzle. x denotes the axial position and $x = 0$ corresponds to the inlet of the combustor. M^{in} and M^{out} are the flow Mach numbers at the nozzle inlet and outlet respectively. It can be seen that if the outflow Mach number M^{out} is greater or equivalent to 1.0, the outlet is choked. And the axial position corresponding to $M_{out} = 1$ can be calculated from Eq. (1).

3. Transient growth of flow disturbances

To characterize the transient growth of flow disturbances in a thermoacoustic system, it is necessary to define an energy measure. When the mean flow is present, the entropy disturbance can be induced. In Ref. [12] the transient growths of two energy measures are investigated. One is associated with both entropy and acoustic fluctuations. The other measure considers the acoustic fluctuations only. In the present paper we also choose and compare these two measures like in Ref. [12]. Detailed analysis is described below.

3.1. The entropy and acoustic fluctuations in flow

To determine the transient growth behavior of a thermoacoustic system, the flow disturbances should be properly described first. The flow parameters are given as axial velocity u, pressure p, density ρ, speed of sound c and Mach number M. The flow parameters consist of a mean part denoted by an overbar and a