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Quantification of uncertainty propagation due to input parameters for simple heat transfer problems

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ABSTRACT

Propagation of uncertainty through the physical model has been investigated in the present paper by solving two specific simple stochastic problems using the Non-Intrusive Spectral Projection method. The uncertain parameters are described by either a Gaussian or a LogNormal probability distribution function. For each of the problems, the stochastic and the deterministic mean solutions have been compared and the respective confidence intervals have been obtained. For the deterministic problems, the confidence intervals have been estimated using both one-dimensional and multi-dimensional bound methods. From the results it has been observed that the differences between the stochastic and the deterministic mean solutions are apparent only when large uncertainties are introduced in the random variables. For both the specific problems, considered in the present study, the confidence intervals for the stochastic problems have been exactly predicted by the deterministic limits when uncertainty is introduced only in one of the parameters. For more than one uncertain parameters, the multi-dimensional bound method produces better agreement with the stochastic confidence intervals than the one-dimensional bound method. The findings are expected to be applicable to problems in heat and mass transfer with similar characteristics or input–output relations.

1. Introduction

In computational predictions, as well as in experiments, it is important to quantify the accuracy of the results [1–3]. Uncertainty quantification in numerical simulations allows one to set the confidence intervals for the predicted system behavior, which may be very important from an engineering point of view. Owing to the increased computing power, the computational tools nowadays can handle various problems involving multi-physics, e.g., fluid–structure interaction, electro-magneto-hydrodynamics, combustion with and without the presence of porous media, phase-change and multiphase flows, turbulence with various time and space scales, etc. The actual physical models involved in such simulations can have high levels of complexity, and thereby can introduce many sources of uncertainties. The manner, in which the uncertainties in individual modeling interact with each other and influence the final outcome of the simulation, is also not trivial owing to the nonlinear nature of the predictive conservation equations. In general, parametric uncertainties in numerical simulations can arise due to several factors, such as, coefficients in the combustion rate expressions, thermophysical properties, initial and boundary conditions, etc. Nevertheless, most often these parameters are assumed as ideal inputs, leading to well defined deterministic solutions and thereby neglecting the effects of their inherent uncertainties, which may be relevant in some situations. As a result, prediction of uncertainty limit is also of utmost importance along with the mean numerical simulation in order to have better insight in to the practical problem and to form a reliable basis for comparison with the experimental data. It may be mentioned here that uncertainties could also be manifested in the simulated outputs due to the discretization error of the numerical scheme [3]. .This issue is, however, beyond the scope of the present article.

The main purpose of stochastic solutions is to determine the mean (expected) solution of the physical problem and to obtain the solution confidence interval for a given uncertainty in some input parameters. There are several stochastic approaches available those accurately model the uncertainty propagation of the input parameters into the output variables during simulation [4]. The well known Monte Carlo (MC) method can be easily implemented, however, it is computationally expensive, even for a small number

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Nomenclature

$\hat{f}_j$	multi-dimensional spectral mode number *j* of *f*
A	area
$C_{f\,L}$	average skin friction coefficient
f, g	generic functions
h	convective heat transfer coefficient
k	thermal conductivity
L	length
M	uncertain parameter in ODEs
N	number of uncertain parameters
Nu_L	average Nusselt number
P	number of PC expansion terms minus one/perimeter
p	maximum polynomial order in PC expansion
Pr	Prandtl number
Re_L	Reynolds number
S_i	number of samples for uncertain parameter *i*
T	temperature
u	velocity
W	multi-dimensional Gaussian probability distribution
w_i	Gauss-Hermite quadrature weight at point *i*
X	dimensionless space coordinate
x, y	space coordinates

Greek symbols

α	ratio of standard deviation to mean value
η	fin efficiency/similarity variable
γ	dimensional skewness
λ_i	uncertain parameter *i*
λ_n	*n* order one-dimensional spectral mode of λ
μ	mean value/dynamic viscosity
ν	kinematic viscosity
ϕ_i	multi-dimensional orthogonal polynomial number *j*
ψ_n	orthogonal polynomial of order *n*
ρ	density
σ	standard deviation
τ	average shear stress
θ	modified temperature
$\vec{\xi}$	vector of random variables
ξ_i	random variable associated with uncertain parameter *i*

Superscript

$'$	space derivative
$-$	mean quantity
d	deterministic

Subscript

∞	at infinite
b	fin base
c	cross section
i	auxiliary pointer
j, k	auxiliary pointers
n	polynomial order
w	at wall

of uncertain inputs [5,6]. More effective methods are based on the spectral representation of parametric uncertainties, using Polynomial Chaos (PC) decomposition [6–8]. In these methods, the uncertainties are treated as additional dimensions along with time and space and uncertain variables for a given problem are projected on these random dimensions using appropriate PC expansions. A classical PC method is based on the Intrusive Spectral Projection (ISP), which requires reformulation of the governing equations in order to propagate the uncertainty through the model, resulting in a set of equations that are generally coupled and most often require special solvers. Although this approach is effective, however, may not be practically suitable within the context of existing complex nature of the modern in-house or commercial CFD codes those are capable of handling multi-physics problems. An alternative approach could be the Non-Intrusive Spectral Projection (NISP) method, where the expansion coefficients of the stochastic solution are obtained by employing sampling in the deterministic solution space. The NISP approach can be easily applied to almost any deterministic codes, however, as the number of uncertain inputs increases, it requires sophisticated sampling methods to be implemented such that it becomes competitive with the ISP approach [8,9], in terms of CPU cost.

In practical heat transfer and fluid flow problems the analytical solutions are generally rare owing to the various nonlinearities introduced by the multi-physics and complexities associated with the models, and hence, numerical solutions are often inevitable. Moreover, availability of the analytical solution for a given problem does not necessarily ensures the possibility of obtaining analytical solutions to the set of equations, generated by the ISP, in order to describe the stochastic problem. Even when numerical solutions are required, many times they are obtained using commercial codes, which, in general, prevent access to the original source code. All these situations eventually require the use of non-intrusive techniques in order to perform stochastic calculations. The implementation and the effectiveness of both intrusive and non-intrusive techniques have been investigated and documented in several scientific publications. Ghanem [10] applied the intrusive spectral formulation of the stochastic finite element method to the problem of one-dimensional heat conduction in a random medium, where the random material properties were treated with both Gaussian and LogNormal models. They have concluded that their intrusive procedure provided a reliable characterization for the propagation of uncertainty from the thermal properties values. Wan and Karniadakis [11] have employed the multi-element generalized polynomial chaos method in order to investigated subcritical resonant heat transfer in a heated periodic grooved channel by modulating the ow with an oscillation of random amplitude, which was assumed to follow both uniform and Beta distributions. They have concluded that their stochastic modeling approach offers the possibility of designing more effective heat transfer enhancement strategies. Le Maître et al. [12] compared the ISP and NISP methods applied to simulations of natural convection in a 2D square cavity with stochastic temperature distribution prescribed on the cold wall. It was concluded that the NISP, using Gauss-Hermite sampling points, performs well when compared with the ISP. It was also shown that if Latin hypercube sampling is used in the NISP, the accuracy of the results is strongly affected by sampling errors and relatively large number of samples is required to achieve similar accuracy. Recently, Ganapathysubramanian and Zabaras [9] applied a NISP method to various stochastic natural convection problems by using a sparse grid collocation technique for sampling the deterministic solution space. They showed that this method performs even better than the ISP or MC methods, specially for large number of stochastic dimensions. The NISP method was also applied for quantification of uncertainties in reacting flow simulations [5,13], where intrusive methods are generally avoid due to the complexity of the governing equations.

In view of the discussion made so far few comments are now in order: *i*) Stochastic solutions are of utmost importance in variety of engineering problems owing to the several uncertainties in the