International Journal of Thermal Sciences 60 (2012) 244–254

Contents lists available at SciVerse ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Boundary and internal heat source effects on the onset of Darcy–Brinkman convection in a porous layer saturated by nanofluid

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ARTICLE INFO

Article history:
Received 21 November 2011
Received in revised form
23 May 2012
Accepted 25 May 2012
Available online 2 July 2012

Keywords:
Nanofluid
Convection
Brownian motion
Thermophoresis
Porous medium
Brinkman model
Galerkin method
Normal mode method

ABSTRACT

The effect of internal heat source on the onset of Darcy–Brinkman convection in a porous layer saturated by nanofluid is studied. The boundaries are considered to be free–free, rigid–rigid and lower-rigid and upper-free boundaries. The Brinkman–Darcy equation with fluid viscosity different from effective viscosity is used to characteristic the nanofluid motion. The model used for nanofluid includes the effects of Brownian motion and thermophoresis. The linear stability theory is employed and the resulting eigenvalue problem is solved numerically using the Galerkin technique with the Rayleigh number as the eigenvalue. The influence of internal heat source strength, nanoparticle Rayleigh number, modified particle-density increment, modified diffusivity ratio, Lewis number, Darcy number and the porosity on the stability of the system is investigated graphically. It is found that the internal heat source, nanoparticle Rayleigh number, modified diffusivity ratio and Lewis number have a destabilizing effect while Darcy number and the porosity show stabilizing effects on the system.

1. Introduction

The present paper deals with the effect of uniformly distributed internal heat source on the onset of Darcy–Brinkman convection in a porous layer saturated by nanofluid, heated from below and cooled from above. The flow of nanofluid is of great interest in many area of modern science, engineering and technology, chemical and nuclear industries and bio-mechanics. The term nanofluids are colloidal suspensions of nanoparticles with typical dimensions of about 1–100 nm dispersed in a non conducting carrier liquids like water, kerosene, ester and hydrocarbons etc. Nanofluids are not naturally occurring but they are synthesized in the laboratory. The choice of base fluid–particle combination depends on the application for which the nanofluid is intended. In the presence of a mere few percents of nanoparticles, a significant increase of the effective thermal conductivity [1–11]. The enhanced thermal conductivity of a nanofluid together with the thermal dispersion of particles and turbulence induced by their motion contributes to an astounding improvement in the convective heat transfer coefficient which in turn makes the nanofluid a superior heat transfer medium for cooling applications [12,13].

Buongiorno [14] gave the conservation equation of a non-homogeneous equilibrium model of nanofluid. On the basis of the transport equation of Buongiorno [14], the stability of problems of onset of convection in nanofluid has been studied by many authors. Tzou [15,16] studied thermal instability problems of nanofluid using method of eigenfunction expansions and observed that nanofluid is less stable then the regular fluid. Dhananjay et al. [17] studied Rayleigh–Bénard convection in nanofluid for free–free boundaries using Galerkin method and they have also discussed the case of overstability that has not been studied by author Tzou [15,16]. The onset of convection in a horizontal nanofluid layer of finite depth was studied by Nield and Kuznetsov [18]. It is found that the critical Rayleigh number can be reduced or increased by a substantial amount, depending on whether the basic nanoparticle distribution is top-heavy or bottom-heavy, by the presence of nanoparticles. They also obtained that oscillatory instability is possible when nanoparticles concentrate near the bottom of the layer. Nield and Kuznetsov [19] have studied the onset of double-diffusive convection in a nanofluid layer. On using one-term Galerkin approximation, they obtained that the stability boundaries for both non-oscillatory and oscillatory cases. The analytical

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1290-0729/$ – see front matter
http://dx.doi.org/10.1016/j.ijthermalsci.2012.05.011

results for oscillatory stability were limited to the case of large Prandtl number and large nanoparticle Lewis number. Alloui et al. [20] studied the natural convection of nanofluids in a shallow cavity heated from below. They observed that the presence of nanoparticles in a fluid is found to reduce the strength of flow field, this behaviour being more pronounced at low Rayleigh number. Also, the temperatures on the solid boundaries are reduced (enhanced) by the presence of the nanoparticles when the strength of convection is high (low). Thermal instability of rotating nanofluid layer has been studied by Yadav et al. [21]. They observed that for the case of non-oscillatory convection, rotation and the difference on the temperature have stabilizing effects while the combine behaviours of Brownian motion and thermoporesis of nanoparticles creates destabilizing effect. Nield and Kuznetsov [22] has been studied the effect of local thermal non-equilibrium on the onset of convection in a nanofluid.

The stability of problems of the onset of convection in porous media saturated by nanofluids has been investigated by many authors. Nield and Kuznetsov [23] investigated the natural convection for flow in a porous medium saturated by nanofluid using the Darcy model. The extension to the Brinkman model was made by Kuznetsov and Nield [24]. The onset of double-diffusive nanofluid convection in a layer of porous medium was studied by Kuznetsov and Nield [25]. Nield and Kuznetsov [26] studied the effect of vertical through flow on thermal instability in a porous medium layer saturated by nanofluid. Natural convection in a nanofluid saturated rotating porous layer: A nonlinear study has been investigated by Bhadauria and Agarwal [27]. Kuznetsov and Nield [28,29] studied the effect of local thermal non-equilibrium on the onset of convection in porous medium layer saturated by a nanofluid using Darcy and Brinkman model respectively. Bhadauria and Agarwal [30] studied the convective transport in a nanofluid saturated porous layer with thermal non-equilibrium model.

Convection induced by internal heat sources has been widely studied because of its wide range of applications in geophysics [31], astrophysics [32], thermal ignition [33], fire and combustion modelling [34], miniaturization of electronic components [35] etc. In such flows the buoyancy force is incremented due to heat source resulting in modification of heat/mass transfer characteristic. The effect of internal heat source on the onset of convection has been carried out for various types of fluids by many researchers. Natural convection for classical fluid, induced by internal heat generation has been studied both experimentally and numerically. The experimental investigation of natural convection induced by internal generation has been studied by Tasaka et al. [36]. This paper attempted the calibration by a horizontal temperature gradient in a horizontal fluid layer with heat conduction, to quantitatively investigate the temperature field of the internally heated convection. Sparrow et al. [37] studied analytically the problem of thermal instability of an internally heated fluid as well as heated from below, with various boundary conditions and showed that with increasing heat generation rate the fluid is prone to instability. Khalili and Shivakumara [38] have studied the onset of convection in a horizontal porous layer is investigated including the effects of through flow and a uniformly distributed internal heat generation for different types of hydrodynamic boundary conditions. The instability parameter is either a Darcy-external or Darcy-internal Rayleigh number which has been determined numerically using Galerkin technique. Influence of Darcy number on the onset of convection in a porous layer with a uniform heat source was studied by Borujerdi et al. [39] and a smooth monotonic variation in the critical Rayleigh was found. Onset of thermal instability in a low Prandtl number fluid with internal heat source in a porous medium has been studied by Cookey et al. [40]. In that problem the internal heat source is taken as directly proportional to the temperature leading to a sinusoidal temperature gradient in the fluid layer and they observed that the onset of stationary instability is hastened by increasing values of the internal heat generation as well as increments in the Prandtl number. Tasaka and Takeda [41] have been considered the effect of heat source distribution in natural convection. The same problem under the effect of rotation has been studied by Chatterjee et al. [42]. Natural convection in a micropolar fluid with thermal dispersion and internal heat generation has been studied by Ei-Hakiem [43]. They observed that in the presence of thermal dispersion and internal heat source lead to increase the flow. The effect of a quadratic basic state temperature profile caused by constant internal heat generation was addressed by Char and Chiang [44] for Bénard–Marangoni convection and the results can be applied to a number of engineering problems like oil extraction from porous media, crystal growth in space and chemical engineering of paints, energy storage in molten salts, colloids and detergents. Shivakumara and Suma [45] have investigated the effect of through flow and constant internal heat generation on the onset of convection using rigid and perfectly conducting boundaries. Nanjundappa et al. [46] studied the effect of uniform internal heat generation on the onset of Brinkman–Bénard convection in a ferrofluid saturated porous layer. It is found that the effect of increase in the value of internal heat source, magnetic Rayleigh number and nonlinearity of the magnetization parameter is to hasten, while increase in the value of Biot number, the ratio of viscosities and reciprocal of Darcy number is to delay the onset of thermo-magnetic convection in a ferrofluid saturated porous layer. Ziabakhsh et al. [47] studied the analytical solution of non-Newtonian micropolar fluid flow with uniform suction/blowing and heat generation using homotopy analysis method (HAM). On the analytical solution for MHD natural convection flow and heat generation fluid in porous medium was studied by Bararnia et al. [48].

The study of the flow and internal heat generation in nanofluids is of special interest and has many practical applications in manufacturing processes in industry. Effect of heat generation/absorption in thermal convection is significant where there exists high temperature difference between the surface and the ambient fluid. Possible heat generation also alters the temperature distribution; consequently the particle deposition rate in nuclear reactors, electronic chips and semiconductor wafers. The objective of the present paper is to study how the onset criterion on the onset of Darcy–Brinkman convection in a porous layer is affected by boundaries, internal heat source and nanofluid parameters. The nanofluid model proposed by Buongiorno [14] is used. The Brinkman–Darcy model has been considered for the porous medium, while the nanofluid layer incorporates the effect of Brownian motion along with thermophoresis. The boundaries of the porous layer are kept at constant but different temperatures and nanoparticles concentrations. In this paper, an analysis is presented for such a problem when the nanofluid is subjected to a constant volumetric internal heating [36–39,44–46]. This can be accomplished, for example, by external radiation as well as viscous dissipation, or by a neutron flux in a lithium blanket such as occurs in fusion reactors. The consequence of this internal heating in the nanofluid is that the fluid temperature and nanoparticle concentration in the quiescent state become nonlinear with respected to the height and the thermal instability is driven by both internal and external heat sources. Thus, uniform heat source took in present study has its own importance. We employ an extensively validated, six terms Galerkin method to study this problem; because it is well known that there are saturations where a single-term Galerkin method gives a zero effect and six-term Galerkin method produces a non-zero effect and this saturation results when one-term eigenfunction happens to be orthogonal to a function that appears in a residue.