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Evaluation of meshless radial basis collocation method (RBCM) for heterogeneous conduction and simulation of temperature inside the biological tissues

Muhammad Jamil, E.Y.K. Ng*

School of Mechanical and Aerospace Engineering, College of Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798, Singapore

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In this paper a novel Radial Basis Collocation Method (RBCM) has been applied to investigate the heterogeneous conduction and bioheat transfer problem. RBCM is a strong form meshless method which uses Radial Basis Function (RBF) interpolation to obtain the solution of the partial differential equation governing the problem under consideration. RBFs hold many advantages like exponential convergence, less dependence on the dimensionality of the problem, ability to deal with complex geometries and ease of implementation, which can be harnessed to one's benefit. Application of RBF interpolation under the framework of RBCM retains its inherent advantages provided that the errors are controlled appropriately. In this research, RBFs have been utilized to solve the steady state heterogeneous conduction and bioheat transfer problem. Approximation function is developed using inverse multiquadratic (IMQ) radial basis functions (RBFs). RBFs are infinitely differentiable functions and have global support. For heterogeneous problem, application of RBF can however become troublesome because of the nonlocality of the RBFs and errors in the domain, interface and boundary can grow large to make the problem unstable. To obtain the exponential convergence, errors at the boundaries, domain and interfaces need to be controlled. Weighted collocation has been used to overcome this problem and retain the inherent properties of the RBFs. RBCM has been successfully applied to solve strong heterogeneous heat conduction and bioheat transfer problem which shows its validity and effectiveness.

1. Introduction

Accurate measurement of temperature in the biological tissues is very important since most of the thermal therapies like, hyperthermia, hypothermia require precise information about the temperature field inside the tissues and their outcome depends on the final achieved temperature. To accomplish this, many bioheat models like Wulff continuum model [1], Klinger continuum model [2], continuum model of Chen and Holmes [3], Pennes bioheat transfer model [4], and Weinbaum–Jiji bioheat model [5] have been proposed to predict the temperature inside the tissues but Pennes' bioheat model attracted most attention because of its simplicity and accuracy for the most cases [4,6]. To obtain the temperature field inside the body is a complex problem because of the presence of inherent heterogeneous structures like veins, arteries and accompanying blood perfusion. Complexity, nonuniformity and health restrictions make it difficult to use experimental procedures to measure the temperature or the property of interest in vivo. To prevent thermal damage, boundary of the isotherm that represents the critical temperature needs to be known. For clinical purposes the inherent limitation of computerized tomography (CT), ultrasound and magnetic resonance imaging (MRI) make it difficult to capture the boundary of thermal damage. For this purpose numerical simulations offer a relative inexpensive and easy option which can be used effectively for multitude of situations. Traditionally, conventional methods such as Finite Element Method [7,8] (FEM), Finite Difference Method (FDM) [9], Monte Carlo Method (MCM) [10] and Boundary Element Method (BEM) [11] have been used for such simulations. These methods are well established and have been developed over the course of many decades. The common characteristic of all these is that they all require mesh to discretize the domain under consideration. In certain cases, the effort required to create a numerical mesh is more than the calculation of solution itself [8,9,11]. The physics and complexity of the geometries may require extremely fine mesh which could result in very skewed elements leading to large errors which would hinder in obtaining an accurate and stable solution.

In order to overcome the meshing difficulties, various meshless methods have been proposed by researchers [12–18]. Meshless

* Corresponding author. Tel.: +65 6790 4455.
E-mail addresses: muha0176@e.ntu.edu.sg (M. Jamil), MYKNG@e.ntu.edu.sg, mykng@ntu.edu.sg (E.Y.K. Ng).

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methods hold promise for many applications because of their advantageous features over the conventional methods like FEM or BEM. These methods depend on the point cloud data with no interconnection between the points rendering them more suitable for adaptive mesh refinement. Their point dependency makes them easier to deal with the complex geometries and makes them less sensitive to the dimensionality of the problem. Many meshless methods have been proposed over the course of last two decades which can be broadly classified into two main categories namely weak formulation or strong formulation depending on the process followed for obtaining the approximate solution of the governing differential equations. Weak formulations like element free Galerkin method (EFGM), method of fundamental solutions (MFS) [19,20] either require background grid to do the numerical integration of the weak form or need to solve Helmholtz equations which makes them computationally expensive. Strong methods like radial basis collocation method (RBCM) [21,22] deal with the original governing differential equations and point collocation is used to satisfy the governing equations in the domain and on the boundary. Strong form meshless methods are 'truly meshless' because they don't require mesh in any form. Multiquadratics are positive definite radial basis functions (RBFs) and have global support. Kansa et al. [23] first used multiquadratics RBFs for the solution of partial differential equations and since then there have been many developments on the properties and application of RBFs [15,19,20,22,24–28]. In this research RBCM has been utilized to simulate the bioheat transfer problem in the two dimensional domain. Approximation function is developed using inverse multiquadratic (IMQ) radial basis function. The aim of this research is firstly to demonstrate the effectiveness of the RBCM for simulating the heterogeneous heat conduction problem and secondly to apply the said method to bioheat transfer problem for accurate prediction. The research objectives were successfully achieved and the results presented in the later sections of this publication are indicative of the appropriateness of RBCM for the above said problems. For this research only steady state problem is considered which can easily be extended to unsteady case. Unsteady analysis can be carried out by discretizing the time derivative and the problem can be solved at each time step similar to steady problem.

2. Theory of radial basis collocation method (RBCM) on 2D heterogeneous problem

Strong form collocation methods are a form of meshless methods which don't require numerical mesh to solve a problem and no preconditioning is required for the original equations describing a particular phenomenon. They are in strong form as they deal with the original equations describing the phenomenon. For the solution approximation different global RBFs are used. For this research, RBFs known as inverse multiquadratics will be used. For general description of RBCM, let's consider a problem consisting of open domain Ω and boundary $\partial\Omega$.

The problem can be defined as:

$$L^\tau T^\tau = f^\tau \text{ in } \Omega \tag{1}$$

$$B^\tau T^\tau = q^\tau \text{ on } \partial\Omega$$

Here T^τ represents the local static temperature, L^τ denotes the differential operator in Ω, B^τ is the boundary condition operator. f^τ is the source term in the open domain whereas q^τ is related to boundary conditions. τ represents the domain number.

Let $\partial\Omega_g, \partial\Omega_h$ represent the Dirichlet and Neumann boundaries respectively then,

$$\begin{aligned} B^\tau T^\tau &= g^\tau \text{ on Dirichlet boundary} \\ B^\tau T^\tau &= h^\tau \text{ on Neumann boundary} \end{aligned} \tag{2}$$

Let's consider a general 2D problem consisting of two homogeneous materials. Strong heterogeneity of material properties is encountered at the interface between the two materials. The two domains are connected by an interface as shown in Fig. 1.

For domain 1, Ω^1:

$$\begin{aligned} L^1T^1 &= f^1 \text{ in } \Omega^1 \\ B_g^1T^1 &= g^1 \text{ in } \partial\Omega^1 \cap \partial\Omega_g \\ B_h^1T^1 &= h^1 \text{ in } \partial\Omega^1 \cap \partial\Omega_h \end{aligned} \tag{3}$$

For domain 2, Ω^2:

$$\begin{aligned} L^2T^2 &= f^2 \text{ in } \Omega^2 \\ B_g^2T^2 &= g^2 \text{ in } \partial\Omega^2 \cap \partial\Omega_g \\ B_h^2T^2 &= h^2 \text{ in } \partial\Omega^2 \cap \partial\Omega_h \end{aligned} \tag{4}$$

For Interface Γ:

$$\begin{aligned} T^1 - T^2 &= 0 \\ B_h^1T^1 + B_h^2T^2 &= 0 \end{aligned} \tag{5}$$

The temperature solution $\tilde{T}_i$ is calculated separately in each subdomain and can be approximated by evaluating RBFs at collocation points i,e

$$\tilde{T}_i(x) = \begin{cases} \tilde{T}_i^1(x) = g_1^1(x)a_{i1}^1 + g_2^1(x)a_{i2}^1 + \dots g_{N_s^1}^1(x)a_{iN_s^1}^1, & x \in \overline{\Omega}^1 \\ \tilde{T}_i^2(x) = g_1^2(x)a_{i1}^2 + g_2^2(x)a_{i2}^2 + \dots g_{N_s^2}^2(x)a_{iN_s^2}^2, & x \in \overline{\Omega}^2 \end{cases} \tag{6}$$

In Eq. (6) coefficients a_{jI} need to be evaluated at all the collocation points in order to obtain the solution where I contains the source points whereas j represents the collocation point in respective domain. Substituting Eq. (6) into Eqs. (3)–(5), results in an algebraic system of equations which can be easily solved for the coefficients.

For numerical implementation of RBCM, the following steps are involved.

Step 1: Identification of collocation points

For ease of implementation, unique set of points in domains, boundaries and interface are identified for each domain. Let τ

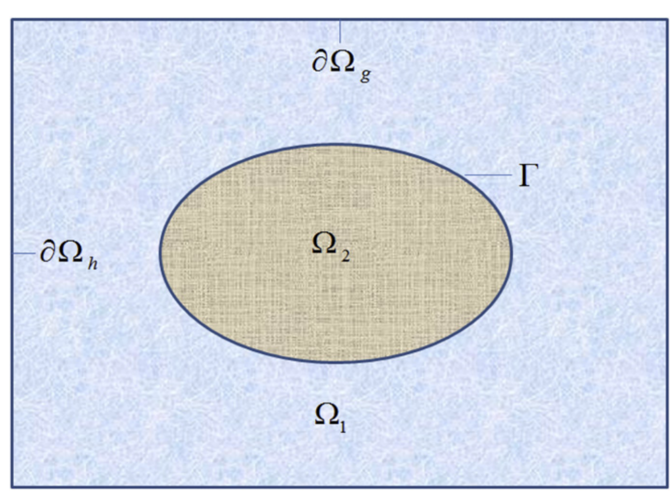

Fig. 1. Heterogeneous domain problem.