International Journal of Thermal Sciences 67 (2013) 87–95

Contents lists available at SciVerse ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Behaviour of laminar plane fountains with a parabolic inlet velocity profile in a homogeneous fluid

N. Srinarayana [a,*], S.W. Armfield [a], Wenxian Lin [b,c]

[a] School of Aerospace, Mechanical and Mechatronic Engineering, The University of Sydney, Sydney, NSW 2006, Australia
[b] School of Engineering and Physical Sciences, James Cook University, Townsville, QLD 4811, Australia
[c] Solar Energy Research Institute, Yunnan Normal University, Kunming, Yunnan 650092, China

ARTICLE INFO

Article history:
Received 15 May 2012
Received in revised form
12 December 2012
Accepted 12 December 2012
Available online 12 January 2013

Keywords:
Plane fountain
Numerical simulation
Scaling
Flapping
Stability
Parabolic velocity profile

ABSTRACT

The behaviour of plane laminar fountains with parabolic velocity inlet profile is studied using numerical simulation over the parametric range $0.25 \le Fr \le 10.0$, $50 \le Re \le 150$ and $Pr = 7, 300, 700$. The behaviour of the flow is most strongly affected by the Froude number and to a lesser extent by the Reynolds number, particularly for weak fountains at low Reynolds numbers. Behaviour is independent of the Prandtl number over the parametric range investigated. Three distinct regimes are observed: a steady symmetric pattern at low Froude numbers ($Fr < 1.25$), an unsteady periodic flapping characterised by lateral oscillations at intermediate Froude numbers ($1.25 \le Fr \le 2.25$) and an unsteady aperiodic flapping at higher Froude numbers. Based on dimensional analysis, the rise height of the fountain is shown to follow the correlation $z_m \sim Fr^{4/3-2/3(c+d)} Re^{-(c+d)} Pr^{-d}$. The constants c and d are determined from numerical results for each regime.

1. Introduction

A fountain, also known as a negatively buoyant jet, is formed whenever a fluid is injected upwards into a lighter fluid, or downward into a denser fluid. In both cases, buoyancy opposes the momentum of the ejected flow until the jet penetrates a finite distance and falls back towards the source. Practical examples of fountain flows are present in natural ventilation, volcanic eruptions, formation of cumulus clouds, and in reverse cycle air-conditioning, to name a few.

In a homogeneous ambient fluid, the fountain behaviour is governed largely by the Reynolds number, *Re*, the Froude number, *Fr*, and the Prandtl number, *Pr*, which are defined at the fountain source as follows,

$$Re = \frac{V_{in} X_{in}}{\nu}, \quad (1)$$

$$Fr = \frac{V_{in}}{\sqrt{g(\rho_{in} - \rho_\infty)/\rho_\infty X_{in}}} = \frac{V_{in}}{\sqrt{g\beta(T_\infty - T_{in}) X_{in}}}, \quad (2)$$

$$Pr = \frac{\nu}{\kappa}, \quad (3)$$

where X_{in} is the radius of the source for a round fountain or the half-width for a plane fountain, V_{in} is the average velocity at the fountain source, g is the acceleration due to gravity, ρ_{in} and T_{in} are the density and temperature of the fountain fluid at the source, ρ_∞ and T_∞ are the density and temperature of the ambient fluid, ν, κ and β are the kinematic viscosity, thermal diffusivity and coefficient of volumetric expansion of the fluid, respectively. The second expression of the Froude number in Eq. (2) applies when the density difference is due to the difference in temperature of the fountain and ambient fluids, using the Oberbeck–Boussinesq approximation.

When the source momentum flux is much higher than that of the buoyancy flux (i.e., when $Re > 1000$ and $Fr \gg 1$), the fountain becomes turbulent close to the source. A combination of a large velocity difference between the interacting fluids, as well as a low resistance to flow, due to a small density difference, then causes large scale turbulence within the fountain. As a result instabilities occur in the fountain up-flow which entrains the ambient fluid and together with opposing buoyancy ultimately restricts the further penetration of the fountain fluid when it attains a certain maximum height. Although instabilities are present in turbulent

* Corresponding author. Tel.: +61 2 9351 2272; fax: +61 2 9351 7060.
E-mail address: snag3258@sydney.edu.au (N. Srinarayana).

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http://dx.doi.org/10.1016/j.ijthermalsci.2012.12.006

Nomenclature

a	power index, in (21)
A_c	cross-section area of the source
b	power index, in (21)
b_{in}	buoyancy flux at the source
c	power index, in (21)
C	constant of proportionality, in (5)
C_1	constant of proportionality, in (6)
d	power index, in (21)
D_H	hydraulic diameter of the source
f	non-dimensional flapping frequency
Fr	Froude number
g	acceleration due to gravity
H	computational domain height
L	computational domain width
m_{in}	momentum flux at the source
p	non-dimensional form of P
P	pressure
P_m	perimeter of the source
Pr	Prandtl number
Re	Reynolds number
t	time
T	temperature
T_{in}	temperature at the source
T_∞	temperature at the ambient
u	non-dimensional form of U
U	horizontal velocity
v	non-dimensional form of V
V	vertical velocity
V_{in}	average velocity at the source
V_m	maximum velocity at the source
x	non-dimensional form of X
X	horizontal coordinate
X_{in}	half-width of the source
y	non-dimensional form of Y
Y	vertical coordinate
z_m	non-dimensional form of Z_m
Z_m	maximum fountain height

Greek symbols

β	coefficient of volumetric expansion
θ	non-dimensional form of T
κ	thermal diffusivity
ν	kinematic viscosity
ρ	fluid density
ρ_{in}	fluid density at the source
ρ_∞	fluid density at the ambient
τ	non-dimensional time

fountains they still exhibit the general fountain characteristics, that is, an initial penetration height is reached during the initial development stage until stagnation occurs; after this, the fountain penetration height is unsteady and unpredictable due to entrainment and interaction of the up-flow, down-flow, and ambient fluids.

The most common parameter used in characterising fountain flow behaviour is the maximum fountain penetration height, Z_m, which is normally made dimensionless as

$$z_m = \frac{Z_m}{X_{in}}, \tag{4}$$

where z_m is the dimensionless maximum fountain penetration height. For turbulent fountains ejected from a round source, it has been found that

$$z_m = CFr, \tag{5}$$

where C is a constant of proportionality. Morton [1] obtained $C = 2.06$ for turbulent round fountains based on an analytical solution using the entrainment concept but without taking into account the effect of the down-flow. Abraham [2] improved this analytical solution by incorporating the effect of the down-flow and obtained $C = 2.74$. Turner [3] derived Eq. (5) using dimensional consistency arguments and determined C as 2.46 with a series of experiments using salt-water solution injected upwards into a fresh water tank with Fr in the range $2 \lesssim Fr \lesssim 30$. This value of C has been further confirmed by some subsequent studies, such as those by Ref. [4] for fountains in magma chambers with very high Re in the range of $1700 \le Re \le 2.7 \times 10^6$, Ref. [5] for $10 < Fr < 300$ and $Re \sim 2000$, and Ref. [6] for $Fr \gtrsim 3$. However, some other values of C have also been obtained experimentally. For example, $C = 2.35$ was obtained by Refs. [7,8] for $3 \le Fr \le 257.7$ and $870 \le Re \le 2710$; $C = 2.1$ was given by Ref. [9] for $15 \le Fr \le 78$ and $1250 \le Re \le 7500$; $C = 3.06$ was obtained by Ref. [10] for $Fr > 7$. Some of the past investigations on turbulent round fountains were further summarized in several recent studies, such as Refs. [11–18].

On the other hand, if the discharge momentum flux of a fountain flow plays a less important role than the negative buoyancy flux, the flow will be in the laminar/transitional region. For these weak fountains with small Fr values, it has been shown that their flow behaviour is considerably different to that of turbulent fountains (see, e.g., Refs. [6,10,12,18–22]). As the current work deals with plane fountains, the interested reader is referred to Refs. [6,10,12,18–20,23–25] for more detailed information on the round fountain behaviour.

The behaviour of plane fountains, which are formed by injecting a denser fluid into a homogeneous light ambient fluid from a long narrow slot, has also been investigated, although apparently not so extensively as round fountains. For a turbulent plane fountain, Campbell & Turner [4] and Baines et al. [5] showed that dimensional consistency requires that z_m has the following scaling,

$$z_m = C_1 Fr^{4/3}, \tag{6}$$

where C_1 is another constant of proportionality. Eq. (6) was validated by the experiments carried out by Ref. [4] for $Fr < 200$ and Ref. [5] for $Fr < 500$. Both studies gave $C_1 = 0.65$. However, Bloomfield & Kerr [26] obtained a value of 2.46 for C_1 much larger than 0.65, by using the same experimental rig as used by Ref. [5]. Hunt & Coffey [27] addressed this difference and suggested that the much lower value of C_1 obtained by Ref. [5] was caused by an error made by Baines et al. [5] in their presentation of their experimental data. The studies on turbulent plane fountains in Refs. [4,5] did not involve any effect of mass flux on the small Froude number fountains. Zhang & Baddour [28] conducted a series of experiments on plane turbulent fountains primarily to study the effect of mass flux, momentum flux and buoyancy flux on the properties of plane turbulent fountains for both large and small Froude number in the range $0.6 \le Fr \le 114$ and $325 \le Re \le 2700$. Their experiments for $6.5 \lesssim Fr \lesssim 113$ support the 4/3-power law as represented by Eq. (7). However, for small Froude numbers, they argued that alternative