Slip flow fluid-structure-interaction

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ABSTRACT

While many microscale systems are subject to both rarefaction and fluid-structure-interaction (FSI) effects, most commercial algorithms cannot model both, if either, of these for general applications. This study modifies the momentum and thermal energy exchange models of an existing, continuum based, multifield, compressible, unsteady, Eulerian-Lagrangian FSI algorithm, such that the equivalent of first-order slip velocity and temperature jump boundary conditions are achieved at fluid-solid surfaces, which may move with time. Following the development and implementation of the slip flow momentum and energy exchange models, several basic configurations are considered and compared to established data to verify the resulting algorithm's capabilities.

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1. Introduction

Both rarefaction and fluid-structure-interaction (FSI) effects are significant for many microscale systems. Examples include micro valves, pumps, actuators, particulate flows, porous flows, two-phase flows, micro-air-vehicles, combustion, and heat exchangers. Rarefaction, typically quantified by the Knudsen number, Kn, which is the ratio of the fluid's mean free path to the characteristic length of the system, becomes significant for gaseous systems at the microscale. Rarefaction results in discontinuities of the velocity and temperature at fluid-solid boundaries, which, for the slip flow regime, $0.01 \leq Kn \leq 0.1$, are typically modeled with first-order slip velocity [1] and temperature jump [2] boundary conditions applied to the continuum based conservation of momentum and energy equations, respectively. FSI effects are significant for any system in which the thermal-fluid and structural dynamics are coupled, and consequently cannot be considered independently. As listed previously, there are already many microsystems that operate with FSI effects; and, as many microfabrication technologies evolve toward the use of more flexible materials than those historically used in the microelectronics industry, such as with printing and molding [3,4], FSI effects may become an even more significant microsystem design consideration. However, while there are many microscale systems that are subject to both rarefaction and FSI effects, currently available computational algorithms do not, typically, have the ability or versatility required to accurately model

these effects for a generic microsystem, and as a result, there are few studies which have considered FSI for microsystems [5–20], and no widely available studies that have numerically considered both FSI and rarefaction in a microsystem.

The primary objective of this study is to modify the momentum and energy exchange models of an existing FSI algorithm, such that the equivalent of first-order slip velocity and temperature jump boundary conditions are achieved at fluid-solid boundaries, which may move and deform arbitrarily with time. The FSI algorithm that is utilized is a three-dimensional, unsteady, continuum based Eulerian-Lagrangian methodology in which fluids, modeled using ICE (implicit, continuous fluid, Eulerian) and solid materials, modeled with MPM (the material-point-method), may be modeled either independently or simultaneously. ICE is a finite volume, cell-centered, multimaterial, compressible, computational fluid dynamics (CFD) algorithm that originated at Los Alamos National Laboratory [21,22]. And, MPM is a particle based method for solid mechanics simulations [23,24]. The development and documentation of the MPM-ICE implementation currently utilized is given in [25–28]. The MPM-ICE FSI algorithm utilizes a statistically averaged, or 'multifield,' approach, where, each material is continuously defined (ρ, $\mathbf{u}$, e, T, v, θ, σ, P), with some probability, over the entire computational domain. This approach differs from the perhaps more common, separate domain methodology, in which fluid and solid materials are defined separately, with only one material at each point, and interaction only occurring at material boundaries. The multifield approach is advantageous for the current application, because it tightly couples fluid-structure-interactions through the conservation equations, rather than explicitly though specified boundary conditions, which allows arbitrary distortion of material

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Nomenclature

A	cell fluid-solid surface area
A_c	fin cross sectional area
Bi_D	Biot number, hD/k_s
C	Stokes drag coefficient, $C_D Re/2$
c	speed-of-sound
C_D	drag coefficient, $F_D/(1/2)\rho_\infty u_\infty^2 DL$
cfl	Courant–Friedrichs–Lewy number
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
D	diameter or characteristic length
e	internal energy per unit mass
E	Young's modulus of elasticity, $9GK/(G+3K)$
$\boldsymbol{f}$	force per unit volume
F_D	drag force
Fo	Fourier number, $\alpha t/D^2$
G	shear modulus of elasticity
H_{rs}	thermal energy exchange coefficient
H_{rs}^{slip}	slip flow thermal energy exchange coefficient
h	heat transfer coefficient or channel height
I	moment of inertia
K	bulk modulus of elasticity
K_{rs}	momentum exchange coefficient
K_{rs}^{slip}	slip flow momentum exchange coefficient
k	thermal conductivity
Kn	Knudsen number, λ/D
L	length
L_2	norm, $\|x\|_2 = \sqrt{\sum_{i=1}^n x_i^2}$
Ma	Mach number, $ReKn\sqrt{2/(\pi\gamma)}$
N	number of materials
Nu	Nusselt number, hD/k
P	pressure
p	order-of-accuracy, $\log((\Phi_{2\Delta x} - \Phi_{4\Delta x})/(\Phi_{\Delta x} - \Phi_{2\Delta x}))/\log(2)$
Pr	Prandtl number, $c_p\mu/k$
Q	rotation matrix
q	thermal energy exchange rate per unit volume
q''	heat flux
R	gas constant
Re	Reynolds number, $\rho u D/\mu$
RMS	root-mean-square, $x_{rms} = \sqrt{(1/n)\sum_{i=1}^n x_i^2}$
T	temperature
t	time
T_b	fin base temperature
$\boldsymbol{u}$	velocity vector
V	cell volume
$\boldsymbol{x, y, z}$	Cartesian coordinate directions
$\boldsymbol{x', y', z'}$	surface coordinate directions

Greek symbols

α	thermal diffusivity, $k/\rho c_p$
β_t	first-order temperature jump coefficient, $[(2-\sigma_t)/\sigma_t][2\gamma/(1+\gamma)][k/(c_p\mu)]$
β_v	first-order velocity slip coefficient, $(2-\sigma_v)/\sigma_v$
γ	ratio of specific heats, c_p/c_v
δ	fin deflection
θ	volume fraction
Θ	nondimensional temperature, $(T-T_\infty)/(T_b-T_\infty)$
λ	mean free path, $\mu/\rho\sqrt{2RT/\pi}$
μ	dynamic viscosity
ν	kinematic viscosity, μ/ρ
ρ	density
$\boldsymbol{\sigma}$	stress
σ_t	thermal accommodation coefficient
σ_v	momentum accommodation coefficient
τ	shear stress
υ	specific volume
ϕ	rotation angle about the $\boldsymbol{z}$-axis
Φ	numerical result
ψ	rotation angle about the $\boldsymbol{y}$-axis

Subscripts

∞	freestream value
i	inlet value
m	mean value
o	outlet value
r	material index
s	material index
w	wall value
x, y, z	Cartesian coordinate directions

Superscripts

-	before exchange contribution
+	after exchange contribution
0	initial value

and material surfaces without explicit surface tracking, passing of boundary conditions, and excessive stability and convergence issues. Use of the MPM-ICE algorithm to evaluate rarefaction with FSI is further merited, as rarefaction effects have already been successfully modeled utilizing the independent CFD (ICE) portion of the algorithm, with slip boundary conditions implemented at the computational domain boundaries [29–31].

The multimaterial governing conservation equations employed by the MPM-ICE algorithm, without effects that are not considered in the present research (chemical reactions, turbulence, multiphase Reynolds stress, gravity, etc.), are given in Eqs. (1)–(3) [28].

$$\frac{\partial \rho_r}{\partial t} + \nabla \cdot (\rho\boldsymbol{u})_r = 0 \qquad (1)$$

$$\frac{\partial(\rho\boldsymbol{u})_r}{\partial t} + \nabla \cdot (\rho\boldsymbol{uu})_r = -\theta_r\nabla P + \nabla \cdot (\theta\tau)_r + \sum_{s=1}^N \boldsymbol{f}_{rs} \qquad (2)$$

$$\frac{\partial(\rho e)_r}{\partial t} + \nabla \cdot (\rho e\boldsymbol{u})_r = -\frac{P\theta_r}{\upsilon_r}\frac{d\upsilon_r}{dt} + (\theta\tau)_r : \nabla\boldsymbol{u}_r + \nabla \cdot (\theta k\nabla T)_r + \sum_{s=1}^N q_{rs} \qquad (3)$$

Equations (1)–(3) are the ensemble average, r-material, conservation of mass, momentum, and energy equations respectively, where there are N materials, θ_r is the r-material volume fraction, and $\sum_{s=1}^N \boldsymbol{f}_{rs}$ and $\sum_{s=1}^N q_{rs}$ are models for the momentum and energy exchange between materials. Eqs. (1)–(3), along with individual material constitutive or equation-of-state models, and models for $\sum_{s=1}^N \boldsymbol{f}_{rs}$ and $\sum_{s=1}^N q_{rs}$, form a complete system of equations. The detailed numerical solution strategy utilized by the MPM-ICE algorithm to solve this system of equations is presented in [28], and consequently will not be duplicated here. However, in a few words, the numerical approach involves operator splitting. For each timestep, the quantities on the right-hand-side of Eqs. (1)–(3) are computed first - this is the Lagrangian phase of the timestep. The conserved quantities, that is, mass, momentum, and energy, for