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Configurations for single-scale cylinder pairs in natural convection

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A thermal-fluid system consisting of an isothermal heated cylinder pair cooled naturally in air is examined. The single-scale cylinder pairs are horizontally orientated and investigated in both horizontal and vertical alignment to determine if optimal configurations exist which maximize heat dissipation at the local cylinder and global array levels. Numerical and experimental approaches are used to assess cylinder interactions in the laminar flow regime $10^4 \leq Ra_D \leq 10^5$. Experimental measurements and numerical predictions confirm the scaling relationships determined using scale analyses and intersection of asymptotes methods. The optimal horizontally aligned configuration was found when the separation distance resulted in merging of the thermal boundary layers of adjacent cylinders, hence confirming the constructal approach for this geometry and flow regime. Similar separation distance result from combined objectives of maximizing local cylinder and overall array heat transfer performance. In contrast, vertically aligned pairs have multiple constructal configurations and depend on the significance of internal local convective resistances. Upper cylinder Nusselt number is enhanced over a single cylinder when the combined effects of lower cylinder plume velocity and temperature impose a beneficial buoyancy-assisted flow. Through defining the plume characteristics and applying a mixed convection analogy, a prediction of the local and global thermal performances is presented.

1. Introduction

Natural convection from an isolated heated body in both external and internal environments is a classical topic in thermal sciences. Much attention has been given to this area mainly due to the practical significance of understanding buoyancy-induced flows from circular or cylindrical objects immersed in a medium. Applications include passive heat exchangers, solar collectors, and heat generating components in aerospace compartments and electronics equipment.

Over the past decades, research on heat transfer characteristics of circular cylinders in natural convection has been well-documented in the literature [1–3]. The research has also been extended to studies on multiple cylinder arrangements to examine the impact of cylinder interactions [4,5]. Sparrow and Niethammer [4] experimentally assessed a pair of horizontally orientated cylinders for the influence of separation distance and cylinder temperature imbalances on natural convection for Rayleigh numbers of order 10^4–10^5. The cylinder pair was vertically aligned, and it was found that the upper cylinder Nusselt number could be improved by a factor of 1.25 over a single cylinder case for separation distances of 6–9 diameters upstream. However, closer distances of less than 4 diameters resulted in a reduction in the performance of the upper cylinder due to the adverse effects caused by the lower cylinder plume.

Sadeghipour and Asheghi [5] produced similar trends for Rayleigh numbers less than 10^3, and extended the study to vertically aligned cylinder arrays containing 2–8 cylinders. Tokura et al. [6] conducted experiments on vertical arrays containing two, three and five cylinders to determine local and average heat transfer coefficients for Rayleigh numbers spanning 10^4–10^5. Similar magnitude enhancements as described by Sparrow and Niethammer [4] were noted. Using an interferometric technique, the authors also presented isotherms for two different array spacings which highlighted the plume interactions that can result in reduction in heat transfer from downstream cylinders. For a very small spacing of 10% of the diameter, this reduction has been attributed to dead zones above and below the upper cylinders that result from the buoyant plume being unable to penetrate the interstices of the array. Corcione [7] also examined multiple vertically

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Nomenclature

C_P	Specific heat capacity, J/kg K
D	Diameter, m
F_{ij}	View factor from surface i to j
$f'(\eta)$	Velocity profile
g	Gravity, m^2/s
Gr_y	Grashof number $\equiv g\beta y^3(T(y) - T_\infty)/\nu^2$
h	Heat transfer coefficient, W/m^2 K
H	Height, m
$H(x,y)$	Heatfunction
k	Thermal conductivity, W/m K
L	Length, m
$\dot{m}$	Mass flow rate, kg/s
Nu	Nusselt number $\equiv hD/k$
P	Pressure, m
Pr	Prandtl number $\equiv \nu/\alpha$
q	Heat transfer rate, W
q^*	Non-dimensional heat flux density, Eq. (15)
R	Thermal resistance, K/W
Ra_D	Rayleigh number $\equiv g\beta D^3(T_w - T_\infty)/\alpha\nu$
Re_D	Reynolds number $\equiv UD/\nu$
r,θ,z	Cylindrical coordinates, m
S	Cylinder spacing, m
S^*	Non-dimensional spacing $\equiv S/D$
t	Thickness, K
T	Temperature, K
T^*	Non-dimensional temperature $\equiv T - T_\infty/T_w - T_\infty$
u,v	Velocity components, m/s
v^*	Non-dimensional velocity, Eq. (18)
W	Enclosure width/height, m
x,y	Cartesian coordinates, m
x^*	Non-dimensional wall distance $\equiv x/D$
y^*	Non-dimensional wall distance $\equiv y/D$

Greek symbols

α	Thermal diffusivity, m^2/s
β	Coefficient of thermal expansion, 1/K
δ_T	Thermal boundary layer thickness, m
ε	Emissivity
$\phi(\eta)$	Temperature excess profile
η	Similarity variable $\equiv(x/y)\,(Gr_y/4)^{1/4}$
μ	Dynamic viscosity, kg/m s
ν	Kinematic viscosity, m^2/s
ρ	Density, kg/m^3
σ	Stefan–Boltzmann constant, W/m^2 K^4

Subscripts

cd	Conduction
ct	Contact
cv	Convection
D,s	Single cylinder
D,u	Upper cylinder
E	Cylinder ends
ext	External to cylinder
r	Radiation
s	Surroundings
T	Total
w	Cylinder wall
∞	Ambient/cold wall

aligned cylinder arrays using numerical methods across a wide range of Rayleigh numbers of order 10^2–10^5. In this study, the cylinder spacing was varied from 2 to 50 diameters. The parametric study was used to develop numerical correlations for individual cylinders and the overall performance of an array using cylinder Rayleigh number, spacing-to-diameter ratio, and number of cylinders as dependent correlating parameters.

Razelos [8] measured heat transfer and also plume interaction between cylinder pairs with temperature imbalances using interferometry. Reduction in upper cylinder heat transfer for spacings less than 5 diameters were observed and increases occurred above this spacing for a Rayleigh number of 13,000 and without temperature imbalance in the array.

Yuncu and Batta [9] numerically investigated the laminar flow around vertically aligned horizontal cylinders and compared the results favourably with the experimental data of Sparrow and Niethammer [4]. The authors quote the maximum enhancements in cylinder Nusselt number to occur at approximately 9 diameters above the lower positioned cylinder.

Chae and Chung [10] recently investigated the effect of separation distance for vertically aligned cylinders in high Prandtl number fluids between 2014 and 8334, and Rayleigh numbers spanning 10^7–10^{10}. Using an analogy with mass transfer measurements, the authors suggest that local heat transfer is increased at the bottom surface of the upper cylinder due to mixed convection effects whereas the top surface heat transfer is decreased. Relaminarization on the top surface of the upper cylinder was noted for the turbulent flow regime which was also attributed to a local reduction in thermal performance.

Farouk and Guceri [11] presented the complex flow interactions that occur in natural convection of cylinders in single rows, double rows in-line, and double rows staggered. A numerical approach for solving both laminar and turbulent flow regimes was used and a symmetry unit was implemented to replicate the scenario of a large number of cylinders within an array. The influence of horizontal and vertical separation distances on cylinder Nusselt number was assessed with enhancements and degradations observed depending on these distances.

Previous studies [4–12] suggest that an optimum separation distance for the vertically aligned arrangement exists. However this outcome is typically based on Nusselt number of upper cylinders or average cylinder Nusselt numbers of the array. In both situations, this optimum spacing ignores the impact on volumetric heat transfer from the array due to variation in separation distance. This is an important consideration for compact designs with optimal global performance, particularly for vertically aligned configurations as the majority of studies using air quote maximum performance at spacings of order 10 diameters. Indeed one of the earliest studies on cylinder arrays by Marsters [12] concluded that a design compromise may be necessary, as upper cylinder enhancements of 30% are achieved at large spacings which adversely impacts the total array dimensions. Despite this inherent characteristic throughout vertical array studies, none of the previous works on vertically aligned cylinders have examined the combined effects of local cylinder enhancements with overall global performance in a given volume.

Bejan et al. [13] used a more appropriate metric of total heat flux density of the array to describe the optimal configuration for staggered horizontal cylinders. The global performance of the array was the primary objective of the study, and as a result local cylinder Nusselt numbers within the array were absent. The authors imposed the conditions of the analysis as for staggered single scale cylinders with centres forming equilateral triangles. It was demonstrated that the optimum spacing had power law