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The effect of thermal radiation on the natural convection boundary layer flow over a wavy horizontal surface

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ABSTRACT

In this article, natural convection boundary layer flow is investigated over a semi-infinite horizontal wavy surface. Such an irregular (wavy) surface is used to exchange heat with an external radiating fluid which obeys Rosseland diffusion approximation. The boundary layer equations are cast into dimensionless form by introducing appropriate scaling. Primitive variable formulations (PVF) and stream function formulations (SFF) are independently used to transform the boundary layer equations into convenient form. The equations obtained from the former formulations are integrated numerically via implicit finite difference iterative scheme whereas equations obtained from lateral formulations are simulated through Keller-box scheme. To validate the results, solutions produced by above two methods are compared graphically. The main parameters: thermal radiation parameter and amplitude of the wavy surface are discussed categorically in terms of shear stress and rate of heat transfer. It is found that wavy surface increases heat transfer rate compared to the smooth wall. Thus optimum heat transfer is accomplished when irregular surface is considered. It is also established that high amplitude of the wavy surface in the boundary layer leads to separation of fluid from the plate.

1. Introduction

Recently, radiative-convective heat transfer flows have received noticeable attention due to its applications in high temperature aerodynamics (see Ref. [1]). Smooth surfaces transfer less heat energy as compared to the rough or irregular surfaces. Perhaps, surfaces are intentionally made non-uniform or uneven in order to acquire enhancement in the rate of heat transfer. Flow over the rough surfaces is common in industries. Therefore, knowledge about heat transfer through irregular surfaces becomes important in this context. Solar collectors, condensers in refrigerators, cavity wall insulating systems, grain storage containers, industrial heat radiators, for example, are a few of many applications where rough surfaces are encountered to transfer small or large scale heat (see Ref. [2]).

In this context, Yao [3] and Moulic and Yao [4] initially focused on the effects of nonuniform surfaces for the boundary layer natural convection flow. Hossain and Rees [5] presented numerical solution of combined heat and mass transfer in natural convection flow from a vertical wavy surface and obtained the results for extensive range of Schmidt number, *Sc* (ranging from 7 to 1500). Problems of natural and mixed convection flow over the vertical wavy surface have been considered respectively by Jang et al. [6] and Jang and Yan [7]. The role of thermal radiation on an uneven surface for mixed convection flow was reported by Molla and Hossain [8]. Quite a number of techniques were used in their analysis to interpret results in the form of shear stress, heat transfer rate and velocity and temperature distributions. More interestingly, flow over a complex wavy surface was considered by Molla et al. [9]. This complex wavy surface was oriented in the vertical direction and composed with the help of two sinusoidal functions. Authors expressed the numerical results in the form of tangential and normal velocity components, isotherms, streamlines and temperature distribution.

It is found from the literature that convection from wavy horizontal plate has not been studied as extensively as the case of vertical wavy surface. In view of this, Rees and Pop [10] presented an analysis on free convection flow induced by a horizontal wavy surface in a porous medium. In their study, focus was given on those cases where the waves have an $O(Ra^{-1/3})$ amplitude, where

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Nomenclature

$\overline{a}$	Rosseland mean absorption coefficient
g	acceleration due to gravity
Gr	Grashof number
L	characteristic length scale associated with the waves
N	maximum number of nodes along the ξ-direction
$\overline{p}$	dimensional pressure
P	dimensionless pressure
Pr	Prandtl number
q	dimensionless heat transfer
q_r	radiative heat flux
R_d	thermal radiation parameter
$\overline{T}$	dimensional temperature of the fluid in the boundary layer
T_∞	temperature of the ambient fluid
T_w	temperature at the surface
$\overline{u}, \overline{v}$	dimensional fluid velocities in the $\overline{x}$– and $\overline{y}$–directions respectively
u, v	dimensionless fluid velocities in the x- and y-directions respectively
$\overline{x}, \overline{y}$	dimensional Cartesian coordinates
x, y	dimensionless Cartesian coordinates

Greek symbols

$\overline{\alpha}$	thermal diffusivity
A	dimensional amplitude of wavy surface
α_0	dimensionless amplitude of wavy surface
β	volumetric coefficient of thermal expansion
η	similarity variable
κ	thermal conductivity
μ	dynamic viscosity
ν	kinematic viscosity of the fluid
ϕ	phase shift parameter
ρ	density of the fluid
σ_b	Stephan–Boltzmann constant
σ_s	scattering coefficient
τ	dimensionless shear stress
θ	dimensionless temperature function
θ_w	surface temperature parameter ($\theta_w = T_w/T_\infty$)
ξ	streamwise coordinate

Subscript

w	wall condition
∞	ambient condition
i	i-designates the grid point along the η-direction
j	j-designates the grid point along the ξ-direction

the Rayleigh number, Ra, is based on the wavelength of the waves and is assumed large. Later the problem posed by Rees and Pop [10] has been extended by Narayana et al. [11] for the case of double diffusive convection and cross diffusion effects on a horizontal wavy surface in a porous medium. Further, Hossain and Pop [12] investigated MHD flow over a continuous moving horizontal wavy surface. They showed that the flow and heat transfer characteristics are substantially altered by both the magnetic field parameter and the amplitude of the wavy surface. Latterly, Siddiqa and Hossain [13] investigated natural convection flow over a horizontal wavy surface for pure fluid. Coordinate transformation is used to transform the boundary layer equations into an appropriate form which was further utilized to obtain solutions numerically.

It is well known that generally thermal radiation play its role in the enhancement of surface heat transfer particularly in situations where convective heat transfer coefficients are small. A number of mathematical models are used in literature which either introduces thermal radiation heat flux in the energy equations or sometimes in the boundary conditions. Some models even bring in separate governing equations for thermal radiation. Özişik [14], Sparrow and Cess [15] and Arpaci [16] initially studied the interaction of thermal radiation. Later, considering the Rosseland diffusion approximation, investigations on the natural convection flow as well as on the mixed convection flow of an optically dense gray viscous fluid past or along heated bodies of different geometries, such as, vertical and horizontal flat plate, cylinder, sphere, wavy surface and axisymmetic rotating and non-rotating bodies under different boundary conditions have been accomplished by several authors [8,17–22]. In these analyses consideration has been given to gray gases that emit and absorb but do not scatter thermal radiation. A more recent analysis on natural convection flow of thermally radiating fluid along a heated vertical wavy surface has been conducted by Siddiqa et al. [23]. In Ref. [23], authors considered surface radiation and solved the problem with numeric simulations.

It has been reported in literature that rough surface is capable of promoting heat transfer rate as compared to rigid flat plate. From the current analysis, it is expected that sinusoidal surface together with thermal radiation will promote more heat near the wall. Perhaps, this is the extension of the recent analysis of [13], which only consider uneven surface to raise heat transfer. With this motivation natural convection flow is modeled in order to notice thermal radiation effects along an irregular horizontal surface (wavy). Such flows find their applications in high temperature aerodynamics, fire dynamics, glass manufacturing, furnace technology and many other industrial based technologies. Coordinate transformation known as primitive variable formulations as well as stream function formulations are employed to transform the boundary layer equations into parabolic partial differential equations. System of equations obtained from PVF is integrated numerically by employing the implicit finite difference method that uses Thomas algorithm as a solver while on the other hand equations from SFF are solved through Keller-box method. Numerical results are obtained in terms of shear stress and rate of heat transfer for few parameters that occur while addressing the issue.

2. Mathematical formulation

Consider a steady state two dimensional boundary layer flow of an unbounded viscous gray fluid over a semi-infinite heated wavy horizontal surface. The schematic diagram and coordinate system of the wavy pattern following (1) is shown in Fig. 1. It is assumed that the surface temperature of the wavy plate T_w is greater than

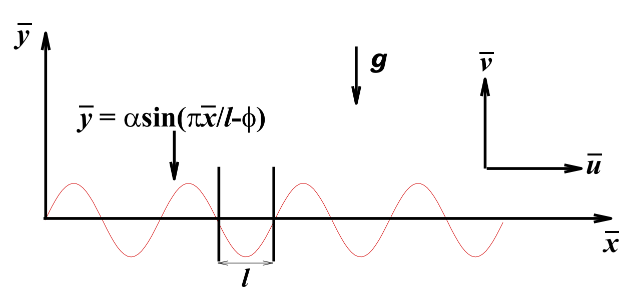

Fig. 1. Physical model and the coordinate system.