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Natural convection along a vertical isothermal plate with linear and non-linear Rosseland thermal radiation

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ABSTRACT

The steady laminar natural convection along a vertical isothermal plate with linear or non-linear Rosseland radiation is investigated in this paper. The problem is self-similar and the results are obtained with the direct numerical solution of the governing equations. This problem is governed by the Prandtl number, the temperature parameter as well as the radiation parameter, and the influence of these parameters on the results are presented in tables and figures. A new radiation parameter is introduced which leads to an asymptotic state. It is found that the temperature profiles take a special S-shape form with an inflection point. Moreover, it is shown that when the wall shear stress increases the wall heat transfer decreases and vice versa.

1. Introduction

Heat transfer by simultaneous radiation and convection has applications in numerous technological problems, including combustion, furnace design, nuclear reactor safety, fluidized bed heat exchangers, solar ponds, solar collectors, turbid water bodies, photochemical reactors and many others [15]. In an optically dense medium radiation travels only a short distance before being scattered or absorbed. The local intensity results from radiation at the nearby surroundings only. For this situation it is possible to transform the integral relation for the radiative energy into a diffusion relation, like the one for the heat conduction. The diffusion approximation provides substantial simplification of the problem ([25]; page 632). The most known diffusion approximation is that of [23].

Probably, the first work on the effect of radiation in a boundary layer flow is that of Smith [26]. Viskanta and Grosh [27] considered the effects of thermal radiation on the temperature distribution and the heat transfer in an absorbing and emitting medium, flowing over a wedge (Falkner–Skan flow), using the Rosseland approximation. Calculations were presented for a fluid with a Prandtl number of 1 and different values of the radiation parameter. As Viskanta and Grosh [27] noted "thermal radiation becomes an additional factor in hypersonic flight, missile reentry, rocket combustion chambers, power plants for interplanetary flight and gas-cooled nuclear reactors".

Afterward, a huge number of papers have been published in the literature on the influence of radiation in boundary layer flows. The open literature is now very rich including cases with moving plates, suction or injection, magnetohydrodynamics, porous media, viscous dissipation, thermophoresis, variable fluid properties, chemical reaction, non-Newtonian fluids, micropolar fluids, etc. Two approaches are used in the literature for the simulation of the radiation term. One is the non-linear Rosseland approximation and the other is the linear one which was probably introduced by Ref. [22]. It should be noted that the dimensionless parameter that is used in the linearized Rosseland approximation is only the effective Prandtl number [8], whereas in the non-linear approximation the problem is governed by three parameters, being the classical Prandtl number, the radiation parameter and the temperature parameter. The objective of the present work is to investigate the effect of radiation on the classical natural convection along a vertical isothermal plate, including either linear or non-linear Rosseland approximation. One important work on this field is that of Hossain and Takhar [4] which concerns the influence of radiation on mixed convection along a vertical isothermal plate. This problem is non-similar, whereas the corresponding work on natural convection is similar. Although the mixed convection includes the case of the natural convection as a limiting case, in the present work the natural convection is studied separately, in order to give better

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insight regarding the considered physical problem. Therefore, the present work is an extension of the classical work of natural convection along a vertical isothermal plate [14], to the case of radiation using the Rosseland approximation in an optically thick medium.

2. Problem definition and solution procedure

Consider the flow along a vertical, semi-infinite plate with u and v denoting respectively the velocity components in the x and y directions, where x is the coordinate along the plate and y is the coordinate perpendicular to x. For a steady, two-dimensional flow, the boundary layer equations are

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty) \tag{2}$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

The boundary conditions are:

$$\text{At } y=0: \quad u=0, \quad T=T_w \tag{4}$$

$$\text{As } y\to\infty \quad u\to 0, \quad T\to T_\infty \tag{5}$$

where υ is the fluid kinematic viscosity, k is the fluid thermal conductivity, T is the fluid temperature, T_w is the plate temperature, T_∞ is the ambient fluid temperature, ρ is the fluid density, c_p is the specific heat, and q_r is the radiation heat flux.

The Rosseland [23] approximation, applies to optically thick media and gives the net radiation heat flux by the following expression

$$q_r=-\frac{4}{3a_R}grad(e_b) \tag{6}$$

Here a_R [m^{-1}] is the Rosseland mean absorption coefficient and e_b [W m^{-2}] the blackbody emissive power which is given in terms of the absolute temperature T by the Stefan–Boltzmann radiation law $e_b = \sigma_{SB}T^4$, with the Stefan–Boltzmann constant being $\sigma_{SB}=5.6697\ 10^{-8}$ W m^{-2} K^{-4}.

For a plane boundary layer flow over a hot surface, Eq. (6) of the net radiation heat flux absorbed in the fluid reduces to

$$q_r=-\frac{16\sigma_{SB}}{3a_R}T^3\frac{dT}{dy} \tag{7}$$

Substituting Eq. (7) into Eq. (3), the energy equation becomes ([25]; page 650)

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\left(\alpha+\frac{16\sigma_{SB}T^3}{3\rho c_p a_R}\right)\frac{\partial T}{\partial y}\right) \tag{8}$$

where $\alpha=k/(\rho c_p)$ is the thermal diffusivity.

Equation (8) is non-linear in T and its solution has some difficulties. A significant simplification of the energy equation (8) can be achieved when the temperature gradients within the flow are small. In such cases, the Rosseland formula (7) can be linearized about the ambient temperature T_∞, by simply replacing T^3 in Eq. (8) by T_∞^3. By doing so, equation Eq. (8) becomes

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\alpha+\frac{16\sigma_{SB}T_\infty^3}{3\rho c_p a_R}\right)\frac{\partial^2 T}{\partial y^2} \tag{9}$$

The non-linear Rosseland approximation is represented by Eq. (8), whereas the linear approximation is represented by Eq. (9).

Equations (1) and (2) together with the energy equation either in form (8) or (9), represent a two-dimensional parabolic problem. Such a flow has a predominant velocity in the streamwise coordinate which in the case presented here is the along-plate direction. In this type of flow, convection always dominates the streamwise diffusion. Furthermore, no reverse flow is acceptable in the predominant direction. The solution of this problem in the present work is obtained using a finite difference algorithm as described by Ref. [21]. In order to obtain a complete form of both the temperature and velocity profile at the same cross section, a nonuniform lateral grid is used. In more detail, Δy takes small values near the surface (dense grid points near the surface) and increases along y. A total of 500 lateral grid cells were used. It is known that the boundary layer thickness changes along the x direction. For that reason the calculation domain must always be at least equal to or wider than the boundary layer thickness. In each case, the maximum potential effort was made in order to have a calculation domain wider than the actual boundary layer thickness. This has been achieved by trial and error. It was seen that in cases of thin the calculation domains, the resulting velocity and temperature profiles were truncated. In such cases, wider calculation domains were constructed, in order to capture the entire velocity and temperature profiles. The parabolic (space marching) solution procedure is described analytically in the textbook of Patankar [21] which "remains to this day a model of simplicity and clarity and one of the most coherent explications of the finite volume technique ever written" [1]. The above solution procedure is implicit and unconditionally stable ([28]; page 276), it has been used extensively in the literature and it has also been included in fluid mechanics and heat transfer textbooks (see Refs. [2], p. 364; [28], p. 271; and [13], p. 124). Finally, this method has also been used successfully in a series of papers by the present author [16–20].

3. Results and discussion

The problem is governed by three non-dimensional parameters [4,5,10,11]. These parameters are the Prandtl number, the radiation parameter and the temperature parameter which are defined as

$$\mathrm{Pr}=\frac{\upsilon}{a} \tag{10}$$

$$R_\infty=\frac{16\sigma_{SB}T_\infty^3}{3ka_R} \tag{11}$$

$$\vartheta_w=\frac{T_w}{T_\infty} \tag{12}$$

Important results for this problem are the non-dimensional wall shear stress and the non-dimensional wall heat transfer defined, according to classical natural convection along a vertical isothermal plate ([28], page 324; [24], page 274), as