Effect of liquid–vapor interface area on the evaporation rate of small sessile droplets

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Dinghua Hu, Huiying Wu[*], Zhenyu Liu

Key Laboratory for Power Machinery and Engineering of Ministry of Education, School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200240, China

ABSTRACT

This study is focused on the role of liquid–vapor interface area in the evaporation of small sessile droplets. The evaporation processes of small sessile droplets with different geometric parameters are numerically investigated with the quasi-steady diffusion model. The numerical results show that droplets with the same surface area are approximately equal in the evaporation rate despite the great differences in contact angles and base radii. It is attributed to the fact that the gradient distributions of vapor concentration surrounding droplets with the same surface area tend to be consistent as the distance from droplet surface increases. Based on the numerical results, a new simple expression with no fitting parameter is developed to calculate the evaporation rate of sessile droplets, and it is found that the evaporation rate is proportional to the square root of liquid–vapor interface area. The proposed expression is in good agreement with the expression of Picknett and Bexon and also validated by the experimental data in open literature. Finally, the evaporation cooling effect of small droplets on an isothermal substrate at room temperature is incorporated into the expression using the average temperature of liquid–vapor interface.

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1. Introduction

Evaporation of small sessile droplets is of great importance in a number of applications such as spray cooling [1], ink-jet printing [2], novel optical and electronic materials manufacturing [3], DNA mapping [4], combustion engineering [5], etc. However, a full understanding of droplet evaporation process is still a challenge [6,7] since it involves multidisciplinary interactions [8–10]. Among various issues related to the evaporation of sessile droplets, the evaporation rate is a fundamental one which has attracted a considerable attention in the past decades [6].

Many experimental studies have been reported on the evaporation rate of sessile droplets. Birdi et al. [11,12] studied the evaporation of water droplets and n-octane droplets on glass and Teflon substrates, respectively. By measuring the weight of droplets, they obtained that the mass evaporation rate (dM/dt) was linearly proportional to the base radius. Subsequently, Rowan et al. [13] performed an experiment to measure the changes of mass, contact angle and base radius with time for water droplets on a poly-

(methyl, methacrylate) (PMMA) substrate. They found that the rate of mass loss (dM/dt) was proportional to the droplet height (obtained from the contact angle and base radius) not the spherical radius. Gelderblom et al. [14] conducted an experiment on the evaporation of water droplets on a superhydrophobic substrate. Their results showed that the mass evaporation rate (dM/dt) of a sessile droplet decreased non-linearly with the decreasing contact angle, especially when the contact angle was over $\pi/2$. It is obvious from the above work that the evaporation rate of a sessile droplet is influenced by its geometric parameters (base radius, contact angle, or height). However, the exact relationship between the droplet's shape and evaporation rate is difficult to be obtained experimentally.

Therefore, theoretical and numerical endeavors have been made to predict the evaporation rate of sessile droplets. In related works, the shape of small droplets was normally considered as a spherical cap [7,8,15–17], and the evaporation of droplets was assumed to be a gas diffusion process driven by concentration gradient [18–22]. Rowan et al. [13] developed an approximate expression for the volume evaporation rate (dV/dt) of sessile droplets by assuming that the vapor concentration gradient is radially outward. Bourges-Monnier and Shanahan [23] obtained a self-consistent solution of the vapor concentration gradient and further proposed an approximate model for the droplet volume evaporation rate ($dV/$

* Corresponding author. Tel.: +86 021 34205299.
  E-mail address: whysrj@sjtu.edu.cn (H. Wu).

Nomenclature

c	vapor concentration [kg m^{-3}]
C_{sat}	saturated vapor concentration [kg m^{-3}]
D	diffusivity of vapor in air [m^2 s^{-1}]
g	gravity constant [m s^{-2}]
h	height of the droplet surface [m]
h_0	height of the droplet apex [m]
H	relativity humidity of the air [−]
H_L	latent heat of vaporization [m^2 s^{-2}]
J	local evaporation flux [kg m^{-2} s^{-1}]
k	thermal conductivity [kg m s^{-3} K^{-1}]
l_c	capillary length $=\sigma/\rho g$ [m]
l_s	thickness of the substrate [m]
$n_{l,a}$	unit vector in the normal direction [−]
Pe	Peclet number $= U_a R/D$ [−]
r	radial coordinate [m]
R	base radius [m]
S	surface area [m^2]
t	time [s]
$\tilde{t}$	non-dimensional time [−]

T	temperature [°C]
U_a	characteristic velocity of convection in air [m s^{-1}]
V	droplet volume [m^3]
$\tilde{V}$	non-dimensional volume [−]
z	vertical coordinate [m]

Greek symbols

θ	contact angle [−]
ρ	density of the liquid [kg m^{-3}]
σ	surface tension [kg s^{-2}]

Subscript

a	ambient air
av	average
hs	hemispherical
l	liquid
s	solid substrate
sat	saturated
sp	spherical
∞	far field of ambient air

dt). Hu and Larson [24] assumed that the vapor concentration distribution surrounding the droplet satisfies the Laplace equation and obtained an empirical expression of mass evaporation rate (dM/dt) based on the numerical results. Sefiane et al. [25] proposed a more precise expression for the volume evaporation rate (dV/dt) of sessile droplets incorporating the thermal effects. Considering the influences of fluid motion inside droplets due to the surface Marangoni instability [26], substrate temperature and thermal conductivity, Girard et al. [9,17] and Semenov et al. [8] performed numerical analyses of the droplet evaporation rate. Most of the theoretical and numerical studies [6–9,15–17,24,25,27,28] showed that the volume evaporation rate of droplets can be predicted by the following general expression

$$\frac{dV}{dt} = -2\pi R \frac{D(c_s - c_\infty)}{\rho} f(\theta) \tag{1}$$

where V is the volume of droplet, t is the time, R is the base radius of droplet, D is the vapor diffusivity in air, ρ is the density of droplet, c_s and c_∞ are the vapor concentration at the droplet surface and far field in the ambient air, respectively, and $f(\theta)$ is a function of contact angle θ with $f(\theta) = 1$ at $\theta = \pi/2$.

It is obvious from Eq. (1) that the evaporation rate can only be determined after $f(\theta)$ is solved. By making an analogy between the diffusive concentration field and electrostatic potential field, Picknett and Bexon [15] derived two fitting polynomials for the exact solution of $f(\theta)$. Popov [16] proposed an analytical solution of $f(\theta)$ in a non-closed form, which is applicable in a wide range of contact angles ($0 < \theta < \pi$) but needs a further numerical calculation. Bourges-Monnier and Shanahan [23] developed an approximate and simple solution of $f(\theta)$, however, this solution deviates obviously from the exact solution when $\theta < \pi/4$. Based on the work of Deegan et al. [19,20], Hu and Larson [24] proposed a simple approximate expression of $f(\theta)$, which is only applicable to droplets with the contact angles of $\theta < \pi/2$. Though thermal effects [25], Marangoni stress [8] and nature convection [29] may affect the evaporation process, it is found that these studies usually started with Eq. (1) as a baseline for predicting the evaporation rate. Thus, obtaining a simple and precise expression for $f(\theta)$ in a wide range of contact angles is of great importance.

In addition, almost all the existing expressions [15,16,23,24] for the evaporation rate of sessile droplets were formulated for spherical cap droplets in terms of two geometric parameters: contact angle and base radius, and few of them can be extended to non-spherical cap droplets, such as ellipsoidal cap droplets [30,31] etc. This is because the definition of a spherical cap needs only two geometric parameters, but a non-spherical cap usually needs more [30,32]. In fact, from the view point of evaporation mechanism, the area of liquid–vapor interface (i.e. droplet surface) where the evaporation take places is the most important geometric parameter to the evaporation rate of a sessile droplet in open stagnant air. However, previous studies [12,13,19,23,24] focused more on the role of contact area or contact angle rather than the liquid–vapor area in the evaporation process of sessile droplets.

In this paper, a numerical study is performed on the evaporation rates of small sessile droplets with different geometric parameters (surface area, base radius, contact angle) on an isothermal substrate at the room temperature. The evaporation is considered as a quasi-steady process and the thermal effects are reasonably neglected in the numerical simulations. The distributions of vapor concentration and vapor concentration gradient surrounding the droplets are discussed. Based on the numerical results, the relationship between the liquid–vapor interface area and the evaporation rate is clarified, from which a new simple and precise expression for the evaporation rate of a small sessile droplet is developed. The expression is compared with the available expressions and experimental data in open literature. Finally, the evaporation cooling effect is incorporated into the expression using the average temperature of liquid–vapor interface.

2. Numerical model and method

2.1. Numerical model

In this section, a numerical model is established to study the relationship between the surface area and evaporation rate of small sessile droplets. Fig. 1 shows a small droplet resting on a solid substrate open to the stagnant air. A cylindrical coordinate system with the radial coordinate r and axial coordinate z is adopted to establish the evaporation model of small sessile droplets, and the