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Electrical circuit analogy for heat transfer analysis and optimization in heat exchanger networks

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HIGHLIGHTS

- Electrical analogy for analysis and optimization of heat exchanger networks is introduced.
- Equivalent thermal circuits for heat exchanger networks are depicted.
- Heat exchanger networks are analyzed by thermal circuits with circuitous philosophy.
- A general mathematical method is proposed for heat exchanger network optimization.
- The proposed method provides several conceptual/quantitative optimization criteria.

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ABSTRACT

Electrical circuit analogy is an effective method for the performance analysis of various heat transfer processes, whereas there is no equivalent thermal circuit for heat exchanger networks (HENs). In view of this limitation, and based on the concept of entransy-dissipation-based thermal resistance (EDTR), we introduce an equivalent thermal circuit to represent the heat transfer process in a heat exchanger, and then analyze the temperature variations of all the working fluids in each heat exchanger to establish the equivalent thermal circuits for such three basic layouts of HENs as multiple-loop, series, and parallel. The combination of these equivalent thermal circuits gives the overall equivalent thermal circuit for any HEN consisting of the three basic layouts. Accordingly, the inherent relationships, i.e., the constraint equations, of all the parameters in a HEN are built by circuitous philosophy. Based on these constraint equations together with the Lagrange multiplier method, we propose a mathematical method for the optimization of heat transfer performance in HENs. Finally, as an example, the heat transfer processes in a district heating system is analyzed and optimized by the newly proposed equivalent thermal circuit and the corresponding optimization method to show the applications.

1. Introduction

Heat exchanger network (HEN), a necessary sub-system, is widely applied in most energy utilization systems, ranging from heating, ventilation and air conditioning system [1], thermal management system [2], to petrochemical industry [3]. The performance analysis and improvement of heat transfer processes are common problems in HENs design, which offers a huge potential for energy conservation as well.

In recent years, a large number of methodologies for the optimal design and control of HENs have been developed to make possible both the energy recovery and the cost reduction, which can be categorized into two broad types. One is the synthesis of HENs, where most methods are proposed based on the pinch analysis [4,5] together with one or more mathematical programming methods, including genetic [6–13], simulated annealing [8], neural network [14], harmony search [15], and particle swarm algorithms [16]. In this category, the mass flow rate, the initial and the final temperatures of each working fluid are given in advance, and thus researchers always vary the arrangement of all heat exchangers and change the area of each heat exchanger to reduce the total cost of HEN. The other type of optimal design is the optimization of the structural and the operating parameters for prescribed HENs, e.g. central chilled water systems [17–20], centralized air-conditioning systems [21] and district heating networks [22–24] in buildings. In these cases, the arrangement of HENs is already determined, but the mass flow rate in each pipe, the area of each heat exchanger need to be designed. For this category of optimization problem, researchers usually analyzed the influence of such factors as fluid

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Nomenclature

A	heat transfer area, m^2
c_p	constant pressure specific heat capacity, J kg^{-1} K^{-1}
E	entransy input rate by a thermal-motive, W K
k	heat transfer coefficient, W m^{-2} K^{-1}
m	mass flow rate, kg s^{-1}
Q	total heat transfer rate, W
q	heat transfer rate, W
R	thermal resistance, K W^{-1}; electric resistance, Ω
T	temperature, K
ξ	flow arrangement factor, K W^{-1}
ε	thermal-motive, K; electromotance, V
Φ_h	entransy dissipation rate, W K
U	electric potential, V
I	current, A
G	heat capacity rate, W/K
g	constant heat capacity rate, W/K
K	constant thermal conductance, W/K
L, L'	Lagrange function
α, λ_{ij}, α', λ_{ij}'	Lagrange multipliers

Subscripts

c	cold fluid
h	hot fluid; heat exchanger
m	midpoint
l	left
r	right
p	parallel
s	series
w	water
max	maximum
min	minimum

mass flow rate, working fluid temperature, and pump pressure on the energy utilization efficiency of HENs and thereafter searched the optimal or near optimal operation strategies with the mathematical programming methods. Recently, Chen and his colleagues [25–28] used the concept of entransy dissipation to construct the mathematical relations between the design parameters and requirements for some typical HENs, and then optimized them by employing the conditional extremum method. The aforementioned methodologies all contribute a lot to offer several directions for impressive improvement of the heat transfer performance of HENs. However, due to the complex mathematical expressions, these methods cannot give a simple and conceptual framework to help us understand the internal relations of each parameter in HENs.

Considering there are many situations in heat transfer analysis which can be analyzed more simply by the use of an electrical counterpart, most heat transfer and heat conduction textbooks [29–31] introduced the electrical circuit analogy and defined the concept of thermal resistance in the analysis of one-dimensional, steady-state heat transfer through a composite structure. Heat transfer rate is regarded as a flow, and the combination of thermal conductivity, thickness of material, and area is considered as a resistance to this flow, and the temperature difference is the potential difference for the flow. However, electrical analogy is usually suitable for linearized problems, while HENs are highly combinatorial and nonlinear systems. Therefore, it seems impossible to analyze and optimize the heat transfer performance of HENs based on the electrical analogy.

Aware of the limitation of electrical analogy in HENs optimization, this contribution intends to introduce the equivalent thermal circuit for HENs based on the concept of entransy-dissipation-based thermal resistance [32–35] and propose a mathematical method to optimize the heat transfer performance of HENs. Finally, a typical HEN is taken as an example and optimized by the newly proposed method to show the application.

2. Analysis of a single heat exchanger

Heat exchanger is a fundamental component in any HEN, and thus we first discuss the heat transfer process in a heat exchanger. Fig. 1 shows the sketch of a counter-flow heat exchanger. The hot fluid flows through the heat exchanger and transfers heat to the cold fluid, which flows in the opposite direction through a separating wall. If the heat capacity rates, i.e. the products of mass flow rates and specific heat capacities, of both hot and cold fluids are infinite, their temperatures do not vary during the whole heat transfer process, and the heat transfer rate between the hot and the cold fluids is

$$Q = kA(T_h - T_c), \tag{1}$$

where Q is the heat transfer rate, k is the overall heat transfer coefficient of the heat exchanger, A is the heat transfer area of the heat exchanger, and T_h and T_c stand for the temperatures of the hot and the cold fluids, respectively.

In this case, similar to a one-dimensional steady-state heat conduction process, the thermal resistance of the heat exchanger, R_h, is defined as

$$R_h = \frac{T_h - T_c}{Q}. \tag{2}$$

Using the electrical analogy, we would view the heat transfer process in this heat exchanger as an equivalent thermal circuit shown in Fig. 2, where the heat flow, Q, across the thermal resistance of heat exchanger, R_h, is driven by the temperature difference between the hot and the cold fluids, $T_h - T_c$.

In reality, however, the heat capacity rates of fluids are always finite, and the temperatures of both hot and cold fluids vary during heat transfer processes. Here, if using the logarithmic mean temperature difference (LMTD) method, the heat transfer rate in the heat exchanger is expressed as

$$Q = kA\frac{\Delta T_{\max} - \Delta T_{\min}}{\ln(\Delta T_{\max}/\Delta T_{\min})}, \tag{3}$$

where $\Delta T_{\max}$ and $\Delta T_{\min}$ stand for the maximum and the minimum temperature differences between the hot and the cold fluids on the same side of the heat exchanger, respectively. LMTD indicates the

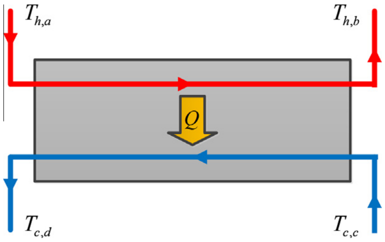

Fig. 1. The sketch of a counter-flow heat exchanger.