Applied Energy 139 (2015) 230–244

Contents lists available at ScienceDirect

Applied Energy

journal homepage: www.elsevier.com/locate/apenergy

Order selection of thermal models by frequency analysis of measurements for building energy efficiency estimation

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HIGHLIGHTS

- The partial differential heat equation is introduced in matrix representation.
- The link between different representations of thermal models is presented.
- Measurable variation of the output is considered for model order reduction.
- Model order reduction can optimize building energy performance characterization.

ARTICLE INFO

Article history:
Received 12 April 2014
Received in revised form 17 November 2014
Accepted 19 November 2014

Keywords:
Heat transfer
Thermal networks
State-space
Frequency analysis
Parameter identification

ABSTRACT

Experimental identification of the dynamic models of heat transfer in walls is needed for optimal control and characterization of building energy performance. These models use the heat equation in time domain which can be put in matrix form and then, through state-space representation, transformed in a transfer function which is of infinite order. However, the model acts as a low-pass filter and needs to respond only to the frequency spectrum present in the measured inputs. Then, the order of the transfer function can be determined by using the frequency spectrum of the measured inputs and the accuracy of the sensors. The main idea is that from two models of different orders, the one with a lower order can be used in building parameter identification, when the difference between the outputs is negligible or lower than the output measurement error. A homogeneous light wall is used as an example for a detailed study and examples of homogeneous building elements with very high and very low time constants are given. The first order model is compared with a very high order model (hundreds of states) which can be considered almost continuous in space.

1. Introduction

Buildings are energy systems which need to be sustainable. There is a necessity to reduce the use of primary energy therefore the estimation of the building energy efficiency is needed. At a local level, this estimation can be used to optimize the operation; at a city level, it can be used for the development of energy policies, contributing to a more sustainable society. In this sense, the energy balance in buildings needs to be modeled to quantify the savings [1–5]. Since the difference between prediction and actual consumption in buildings is important [6], there is a need to develop a methodology that allows the evaluation of the energy performance of buildings based on in-situ measurements. This methodology must be reproducible and its application range must be defined. In this sense, previous studies proposed different models for the analysis of energy consumption in buildings and a distinction is done between solving the direct or the inverse problem [7,8].

It is worth mentioning that all studies, on both direct and inverse problems, use the same basic model, the partial differential equation of heat conduction taking into account the classical principle of energy conservation and Fourier's law of heat conduction [9]. Then, different methods are used to simplify the model or to obtain other model representations, which are particular cases of this model [8,10].

The direct problem, in which the properties of the building materials are supposed known, is mostly solved from principles of heat transfer theory using thermal circuits and looking for criteria which allow determining the best simplification within its theoretical framework [11–17]. The inverse problem, in which

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http://dx.doi.org/10.1016/j.apenergy.2014.11.033
0306-2619/

Nomenclature

A_n	amplitude of a sinusoidal signal
b_k	temperature source on branch k, °C or K
C_l	thermal capacity in node l, J K^{-1}
c	specific heat capacity of the material, J kg^{-1} K^{-1}
d	distance between nodes, m
e_k	temperature difference over the thermal resistance R_k, °C or K
f_l	heat rate source in node l, W
f	frequency, s^{-1}
f_c	cut-off frequency, s^{-1}
$f(t)$	continuous time series
$f_{N_a}(t)$	discrete time series
I_{sv}	vertical global solar irradiance, W m^{-2}
m	number of time samples
M	magnitude, dB
N	number of nodes
N_a	total number of harmonic functions
p	heat rate sources, W m^{-3}
q	heat flux density W m^{-2}
q_k	heat transfer rate on the branch k, W
Q_i, Q_o	inside, outside heat flux density, W m^{-2}
R_k	thermal resistance on the branch of the thermal network k, K W^{-1}
R_{si}^{-1}, R_{so}^{-1}	inside and outside thermal conductance, W K^{-1}
s	complex variable
S	wall surface area, m^{-2}
t	continuous time, s
t_k	discrete time at sample k
T	period, s
T_o, T_i	outdoor, indoor air temperature, °C or K
w	wind speed, m s^{-1}

Greek letters

α	wall absorptivity
ϕ_n	phase, radian
κ	thermal conductivity of the material, W m^{-1} K^{-1}
θ	spatial temperature distribution, °C or K
θ_l	temperature of node l, °C or K
θ_{so}, θ_{si}	temperatures of the outside and inside surfaces, respectively, °C or K
ρ	medium density, kg m^{-3}
τ	time constant, s rad^{-1}
$\partial/\partial t$ or $\dot{\square}$	differential operator in time
$\nabla\cdot$	divergence operator
∇	gradient operator

Vectors and matrices

$\mathbf{A}$	re-arranged incidence matrix of the thermal network
$\mathbf{A}^T$	transpose of the re-arranged incidence matrix
$\mathbf{A}_d$	incidence matrix of the thermal network
$\mathbf{A}_d^T$	transpose of the incidence matrix
$\mathbf{A}_S$	state matrix in the state-space model
$\mathbf{B}_S$	input matrix in the state-space model
$\mathbf{b}$	vector of temperature sources on the branches
$\mathbf{b}_j^c$	j-column vector of the input matrix in the state-space model
$\mathbf{C}$	re-arranged diagonal matrix of thermal capacities
$\mathbf{C}_d$	diagonal matrix of thermal capacities
$\mathbf{C}_S$	output matrix in the state-space model
$\mathbf{c}_i^r$	i-row vector of output vector in the state-space model
$\mathbf{D}_S$	feed through matrix in the state-space model
$\mathbf{E}$	generic matrix
$\mathbf{f}$	re-arranged vector of heat rate sources
$\mathbf{f}_d$	vector of heat rate sources
$\mathbf{f}_C$	vector of heat rate sources connected to nodes with thermal capacity
$\mathbf{f}_0$	vector of heat rate sources connected to nodes without thermal capacity
$\mathbf{G}$	diagonal matrix of thermal conductances
$\mathbf{H}_S$	transfer matrix
H_{ij}	output i regarding to input j component of transfer matrix
$\mathbf{I}$	identity matrix
$\mathbf{u}$	input vector in the state-space model
$\mathbf{x}$	column vector
$\mathbf{y}^T$	transpose of a column vector

Vectors in Greek letters

$\boldsymbol{\theta}$	re-arranged vector of temperatures in nodes
$\boldsymbol{\theta}_d$	vector of temperatures in nodes
$\boldsymbol{\theta}_C$	vector of temperatures in nodes with a thermal capacity
$\boldsymbol{\theta}_0$	vector of temperatures in nodes without a thermal capacity

the properties of the wall are found out from measurements of the boundary conditions, has usually been solved in a similar way by using discretized models and procedures for parameter identification [18–22].

In general, mathematical criteria are used in order to decide on the order of the model. In direct (or simulation) problems, the criteria is mainly based on the comparison of the outputs of the complete (or a very high order) model and the reduced order model. In inverse (or parameter identification) problems, the main idea is to augment the order of the model until the residual (i.e. the difference between the measured output and the output of the model) becomes an uncorrelated white noise [23–28].

In practice, the input signals have a limited domain of variation in amplitude and in frequency. The upper limit of the amplitude is given by the nature of the system; its lower limit is given by the accuracy of the measurement devices. Similarly, the lower frequency of the signals depends on the nature of the system and the higher limit is given by the sampling time. Since the large majority of the physical systems are dissipative, the amplitude gain will be constant at low frequencies and reduced at high frequencies. The frequency at which the reduction of the amplitude gain is so high that it cannot be measured due to the limited accuracy of the devices gives the upper limit of the frequency domain on which the frequency response of the complete model and that of the reduced order model need to be similar.

The present work proposes this criterion for the choice of the order of the model. The structure of the model is obtained from space and time discretization of a continuous model expressed in state-space form [29]. This model will act as a filter for the inputs. The spectral distribution of the input signals reveals the domain on which the two filters, the complete (or the physical process) and the reduced order models, need to give similar results. The objective is to choose the simplest optimal model to be applied in: characterizing the building energy performance, controlling the building energy flow, and forecasting the building energy consumption.

In the next sections, a description of the experimental set-up and of the measured data is presented, then the methodology proposed is stated in its general form and finally it is shown how an adequate structure of the model can be chosen based on frequency analysis of the input signals.