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Viscous dissipation and thermoconvective instabilities in a horizontal porous channel heated from below

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ABSTRACT

A linear stability analysis of the basic uniform flow in a horizontal porous channel with a rectangular cross section is carried out. The thermal boundary conditions at the impermeable channel walls are: uniform incoming heat flux at the bottom wall, uniform temperature at the top wall, adiabatic lateral walls. Thermoconvective instabilities are caused by the incoming heat flux at the bottom wall and by the internal viscous heating. Linear stability against transverse or longitudinal roll disturbances is investigated either analytically by a power series formulation and numerically by a fourth order Runge-Kutta method. The special cases of a negligible effect of viscous dissipation and of a vanishing incoming heat flux at the bottom wall are discussed. The analysis of these special cases reveals that each possible cause of the convective rolls, bottom heating and viscous heating, can be the unique cause of the instability under appropriate conditions. In all the cases examined, transverse rolls form the preferred mode of instability.

1. Introduction

The analysis of the Darcy–Bénard problem in a horizontal fluid saturated porous layer is a classical issue of the studies of stability against thermally-induced convection cells. The importance of the Darcy–Bénard problem and of its several variants stems from the link to the conceptually similar Rayleigh–Bénard problem for a clear fluid. In practice, the interest in the investigation of convective instabilities in a fluid saturated porous material heated from below arises from the several applications either with respect to geophysics, to the hydrology of groundwater, and to the diffusion of chemical contaminants in the soil. A wide literature exists on this subject originated from the pioneering papers by Horton, Rogers [1] and Lapwood [2]. A subsequent extension of this study is Prats problem [3], where a basic horizontal throughflow in the porous layer is assumed, instead of the basic rest state considered in the papers by Horton, Rogers [1] and Lapwood [2]. Comprehensive reviews of this subject, accounting for the wide literature available to date, can be found in Nield and Bejan [4], Rees [5] and Tyvand [6].

Interesting studies about the effect of viscous dissipation on heat transfer and fluid flow in saturated porous media have been published [7–13]. Some of these investigations are devoted to the modelling of the viscous dissipation contribution in the local energy balance [9,12,13]. In particular, Nield [9] discusses the resolution of a paradox arising when both viscous dissipation and inertial effects occur. Breugem and Rees [12] carry out a rigorous volume-averaging procedure for the local balance equations under the assumption of a non negligible viscous dissipation. Several studies have been carried out on the effects of the viscous heating in buoyant flows [7,8,10,11]. For a detailed survey of the wide literature on viscous dissipation in porous media we refer the reader to the book by Nield and Bejan [4], as well as to the recent paper by Nield [13].

Quite recently, the effects of viscous dissipation have been investigated as the possible cause of convective instabilities in porous media [14–19]. In these papers, a fluid saturated porous layer with an infinite horizontal width and a finite thickness is considered. Different flow models and thermal boundary conditions are investigated. Among the cases examined we cite, horizontal basic flow with a bottom adiabatic boundary and a top boundary subject to a third kind condition [14] or with bottom and top adiabatic boundaries [15]. The linear instabilities of the basic horizontal flow of water next to the density maximum state have been studied [16] and the form-drag effects have been included [17]. The Prats problem has been revisited by including both the contributions of viscous dissipation and pressure work in the local energy balance [18]. The case of a basic vertical throughflow with

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Nomenclature

a	wave number, Eq. (32)
A_n, B_n	dimensionless coefficients, Eq. (37)
c	average heat capacity per unit mass
$C_{m,n}$, $D_{m,n}$	dimensionless coefficients, Eq. (50)
Ec	Eckert number, Eq. (9)
$\mathbf{g}$	gravitational acceleration
g	modulus of the gravitational acceleration
Ge	Gebhart number, Eq. (9)
H	channel height
k	average thermal conductivity
K	permeability
L	channel half-width
m, n	integers
Pe	Péclet number, Eq. (19)
$\bar{q}_0$	bottom wall heat flux
Ra	Rayleigh number, Eq. (9)
$\Re\{\}$, $\Im\{\}$	real part, imaginary part
s	L/H, aspect ratio
t	dimensionless time, Eq. (9)
T	dimensionless temperature, Eq. (9)
$\bar{T}_0$	top wall temperature
u, v, w	dimensionless velocity components, Eq. (9)
U, V, W	dimensionless velocity disturbances, Eq. (20)
x, y, z	dimensionless Cartesian coordinates, Eq. (9)

Greek symbols

α	average thermal diffusivity
β	volumetric coefficient of thermal expansion
γ	dimensionless coefficient, Eq. (33)
ε	perturbation parameter, Eq. (20)
η, η_m	Value of B_0, $D_{m,0}$
θ	dimensionless temperature disturbance, Eq. (20)
$\Theta(y)$, $\Theta_m(y)$	dimensionless functions, Eqs. (32) and (46)
λ	$\lambda_1 + i\lambda_2$, complex exponential growth rate
ν	kinematic viscosity
σ	ratio between the volumetric heat capacities of the fluid saturated porous medium and of the fluid
ψ	dimensionless streamfunction, Eqs. (28) and (42)
$\Psi(y)$, $\Psi_m(y)$	dimensionless functions, Eqs. (32) and (46)
Ω	dimensionless parameter, Eq. (54)

Superscript, subscripts

-	dimensional quantity
B	basic flow
cr	critical value
L	longitudinal rolls

viscous dissipation has been considered [19], thus extending the analysis carried out by Homsy and Sherwood [20]. All these investigations confirmed that the effect of viscous dissipation may be the sole cause of the convective instabilities. In other words, linear instabilities induced by the effect of viscous dissipation term may arise even in the absence of a heat input across the bottom boundary. These instabilities are in fact thermoconvective instabilities, although generated internally by the viscous heating and not by an externally impressed temperature gradient.

The aim of the present paper is to develop the above described investigation of the role played by the effect of viscous dissipation on the thermoconvective instabilities in porous media. In the present study, the effect of a lateral confinement due to adiabatic vertical boundaries is considered. Reference is made to a porous channel with an isoflux bottom boundary and an isothermal top boundary. The critical conditions for the onset of either transverse or longitudinal rolls are determined both analytically by a power series method and numerically by a fourth order Runge-Kutta method.

2. Governing equations

We consider the stability of parallel Darcy flow in a rectangular horizontal channel filled with a fluid saturated porous medium. The channel is bounded above and below by two horizontal walls, separated by a distance H, and laterally by two vertical walls separated by a distance $2L$; all walls are impermeable (see Fig. 1). The components of seepage velocity along the $\bar{x}$-, $\bar{y}$- and $\bar{z}$-directions are denoted by $\bar{u}$, $\bar{v}$, and $\bar{w}$ respectively, where the $\bar{y}$-axis is vertical and the $\bar{z}$-axis-axis is directed along the channel. The lower boundary wall $\bar{y} = 0$ is subject to a positive uniform heat flux $\bar{q}_0$, while the upper boundary wall $\bar{y} = H$ is supposed to be isothermal with temperature $\bar{T}_0$. Furthermore, the lateral walls $\bar{x} = \pm L$ are assumed to be adiabatic. Both the Darcy model and the Boussinesq approximation are invoked.

The governing mass, momentum and energy equations can be expressed as

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0, \tag{1}$$

$$\frac{\partial \bar{w}}{\partial \bar{y}} - \frac{\partial \bar{v}}{\partial \bar{z}} = -\frac{g\,\beta\,K}{\nu}\frac{\partial \bar{T}}{\partial \bar{z}}, \tag{2}$$

$$\frac{\partial \bar{u}}{\partial \bar{z}} - \frac{\partial \bar{w}}{\partial \bar{x}} = 0, \tag{3}$$

$$\frac{\partial \bar{v}}{\partial \bar{x}} - \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{g\beta K}{\nu}\frac{\partial \bar{T}}{\partial \bar{x}}, \tag{4}$$

$$\sigma\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{T}}{\partial \bar{z}} = \alpha\left(\frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right) + \frac{\nu}{Kc}\left(\bar{u}^2 + \bar{v}^2 + \bar{w}^2\right). \tag{5}$$

Eqs. (2)–(4) have been obtained by applying the curl operator to both sides of Darcy's law in order to remove the explicit dependence on the pressure field. In Eq. (5), the dissipation function is proportional to the square modulus of the seepage velocity [13].

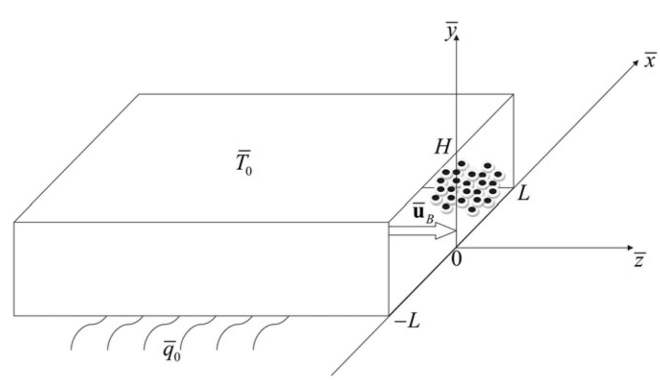

Fig. 1. Drawing of the porous channel.