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Two-dimensional heat conduction in thermodynamics of continua with microtemperature distributions

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ABSTRACT

This paper is concerned with the linear theory of heat conduction in continua with microtemperatures. The work is motivated by increasing use of materials which possess thermal variation at a microstructure level. The theory of plane thermal fields in homogeneous and isotropic bodies is investigated. The first part of the paper is devoted to the basic boundary value problems of the stationary theory. The fundamental solutions of the field equations are established and the potentials of single layer and double layer are introduced. The boundary value problems are reduced to the study of singular integral equations for which Fredholm's theorems hold. Existence and uniqueness results are established. The second part of the paper is devoted to time-dependent problems. First, a solution of Galerkin type of field equations is established. Then a uniqueness theorem and an instability result are presented. The solution of Galerkin type is used to investigate the effects of some concentrated heat sources acting in an infinite medium. The theory is applied to solve the problem of stationary thermal fields in a hollow cylinder.

1. Introduction

The origin of the theories of continua with microstructure goes back to the papers of Ericksen and Truesdell [1], Mindlin [2], Eringen and Suhubi [3] and Green and Rivlin [4]. Mindlin [2] formulated a theory of a continuum which has some properties of a crystal lattice as a result of inclusion of the idea of a unit cell. Mindlin begins with the general concept of a continuum, each material particle of which is a deformable medium. Independently, a theory of microelastic continuum was published by Eringen and Suhubi [3], and a theory of multipolar continuum mechanics by Green and Rivlin [4]. In the theory developed by Mindlin [2], each microelement is constrained to deform homogeneously. In this theory, the spatial coordinates x'_i of the point X' of the microelement Ω are represented in the form $x'_i = x_i + \psi_{ik}\xi_k$, where x_i are the spatial coordinates of the centroid X of Ω, X'_k and X_k are the material coordinates of X' and X, and $\xi_k = X'_k - X_k$. The functions ψ_{ik} are called microdeformations. In the mechanical theory of continua with microstructure the degrees of freedom for each microelement are twelve: three translations, x_i, and nine microdeformations, ψ_{ik}. The theory of continua with microstructure has been studied extensively and an account of the basic results can be found in the works of Truesdell and Noll [5], Kunin [6], Eringen [7] and Mariano [8]. In [9], Grot established the thermodynamics of continua with microstructure when the points of a generic microelement have different temperatures. In this theory the temperature θ' at the point X' of the microelement Ω is a linear function of the microcoordinates ξ_k, of the form $\theta' = \theta + \tau_k\xi_k$, where θ is the temperature at the centroid X. The vector with the components T_k defined by $T_k = -\tau_k/\theta$ is called the microtemperature vector. In the thermomechanical theory of continua with microstructure the unknown functions are x_i, ψ_{ik}, θ and T_k. The theory of continua with microtemperatures has been investigated in many papers (see, e.g., [10–17], and references therein). Recently, a study of heat transfer in nano-fluids has been presented in [18,19]. The structural stability for a rigid body with thermal microstructure has been investigated in [17].

This paper is concerned with the linear theory of heat conduction in homogeneous and isotropic continua with microtemperatures. This work is motivated by increasing use of materials which possess local temperature variation that stems from a local change of structure at the microscopic level (see, e.g., [17–19], and references therein). In the first part of the paper we study the basic boundary value problems of the stationary theory of plane thermal fields. In Section 2 we present the basic equations of the two-dimensional theory of heat conduction in isotropic bodies with microtemperatures. Section 3 deals with the fundamental solutions

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of the field equations. In Section 4 we introduce the potentials of single layer and double layer and reduce the boundary value problems to singular equations for which the Fredholm's basic theorems are valid. Section 5 is concerned with existence and uniqueness results. In the second part of the paper we consider time-dependent problems. In Section 6 we establish a solution of Galerkin type of the field equations. Logarithmic convexity method is used to derive a uniqueness theorem and an instability result. Section 7 is concerned with the harmonic propagation of heat in bodies with microtemperatures. We use the representation established in Section 6 to derive the solutions corresponding to concentrated heat sources acting in an infinite medium. In Section 8 we study the problem of plane thermal fields in a hollow cylinder. The salient feature of the solution is that the temperature field contains new terms characterizing the influence of the microtemperatures and its value is therefore different from the value predicted by the classical theory. In Section 9 we have summarized the original results presented in this paper.

2. Basic equations

In this paper we consider the linear theory of heat conduction in materials with microtemperatures. Let B be a bounded regular region of three-dimensional euclidean space, and let ∂B be the boundary of B. We assume that the material occupying B is homogeneous and isotropic.

The body is referred to a fixed system of rectangular cartesian coordinate frame Ox_i $(i = 1,2,3)$. Throughout this paper Latin subscripts (unless otherwise specified) are understood to range over the integers (1,2,3), Greek indices have the range (1, 2), summation over repeated subscripts is implied and subscripts preceded by a comma denote partial differentiation with respect to the corresponding cartesian coordinate. We denote by $\mathbf{n}$ the outward unit normal of ∂B. Letter in boldface stand for tensors of an order $p \geq 1$, and if $\mathbf{u}$ has the order p then we write $u_{ij\ldots k}$ (p subscripts) for the components of $\mathbf{u}$ in the coordinate frame. If $\mathbf{A}$ is a second-order tensor, then $\mathbf{A}^T$ denotes its transpose.

In the first part of this paper we consider the stationary theory of heat conduction. We deal with functions of position having as their domain of definition the region B. We denote by θ the temperature field over B measured from the constant absolute temperature T_0. Let $\mathbf{T}$ be the microtemperature vector field [9]. The balance of energy can be expressed as

$$q_{j,j} = -S, \tag{2.1}$$

where q_j is the heat flux vector and S is the heat source. We denote by q_{ij} the first heat flux moment tensor. Let us consider a microelement ω of the body. Let x be the center of mass of ω and let z be a generic point of ω. We denote by x_j and z_j the Cartesian coordinates of x and z, respectively, and define S_j by $S_j = x_j - z_j$. For each point z in ω, we associate the vector flux q_j^*. The vector q_j is the surface average of q_j^* and q_{ij} is the surface average of $q_i^* S_j$. The quantities q_j and q_{ji} are associated to the point x. The balance of first moment of energy is given by

$$q_{ji,j} + q_i - Q_i = -G_i, \tag{2.2}$$

where Q_i is the microheat flux average vector, and G_i is the first heat source moment vector. The constitutive equations for isotropic and homogenous bodies are [9]

$$\begin{aligned} q_j &= k\theta_j + k_1 T_j, \; Q_i = (k_1 - k_2)T_i + (k - k_3)\theta_{,i}, \\ q_{ij} &= -k_4 T_{r,r}\delta_{ij} - k_5 T_{i,j} - k_6 T_{j,i}, \end{aligned} \tag{2.3}$$

where δ_{ij} is the Kronecker delta, and k and k_s, $(s = 1,2,\ldots,6)$, are prescribed constants. The Clausius-Duhem inequality implies that [9]

$$\begin{aligned} &k \geq 0, \; 3k_4 + k_5 + k_6 \geq 0, \; k_6 + k_5 \geq 0, \\ &k_6 - k_5 \geq 0, \; (k_1 + T_0 k_3)^2 \leq 4T_0 k k_2. \end{aligned} \tag{2.4}$$

The heat flux q and the heat flux moment vector M_j, at regular points of ∂B, are defined by

$$q = q_j n_j, \; M_k = q_{jk} n_j, \tag{2.5}$$

respectively.

We proceed now to the plane strain problem and for this purpose stipulate that the region B from here on refers to the interior of a right cylinder with the open cross-section Σ. The coordinate frame is chosen in such a way that the x_3-axis is parallel to the generators of B. We denote by L the boundary of Σ (Fig. 1).

Let us assume that the heat sources are independent of x_3 and that $G_3 = 0$ We consider the two-dimensional theory characterized by

$$\theta = \theta(x_1, x_2), \; T_\alpha = T_\alpha(x_1, x_2), \; T_3 = 0, \tag{2.6}$$

where $(x_1,x_2) \in \Sigma$. It follows from equation (2.3) that q_j,Q_k and q_{rs} are all independent of x_3. The constitutive equations become

$$\begin{aligned} q_\alpha &= k\theta_\alpha + k_1 T_\alpha, \; Q_\alpha = (k_1 - k_2)T_\alpha + (k - k_3)\theta_{,\alpha}, \\ q_{\alpha\beta} &= -k_4 T_{\rho,\rho}\delta_{\alpha\beta} - k_5 T_{\alpha,\beta} - k_6 T_{\beta,\alpha}, \end{aligned} \tag{2.7}$$

and $q_3 = 0$ $Q_3 = 0$ $q_{\alpha 3} = 0$, and $q_{33} = -k_4 T_{\rho,\rho}$. The function q_{33} does not appear in the remaining equations and it can be found after the determination of the functions T_α. The equations (2.1) and (2.2) reduce to

$$q_{\alpha,\alpha} = -S, \; q_{\beta\alpha,\beta} + q_\alpha - Q_\alpha = -G_\alpha, \tag{2.8}$$

on Σ. It follows from equations (2.7) and (2.8) that the functions θ and T_α satisfy the equations

$$\begin{aligned} &k\Delta\theta + k_1 T_{\rho,\rho} = -S, \\ &k_6 \Delta T_\alpha + (k_4 + k_5)T_{\rho,\rho\alpha} - k_2 T_\alpha - k_3 \theta_\alpha = G_\alpha, \end{aligned} \tag{2.9}$$

on Σ, where Δ is the Laplacian. The relations equation (2.5) on the regular points of ∂B become

$$q = q_\alpha n_\alpha, \; M_\alpha = q_{\beta\alpha} n_\beta, \tag{2.10}$$

on L, where $n_\alpha = \cos(n_x,x_\alpha)$ and $\mathbf{n}_x$ is the unit vector of the outward normal to L.

To the equation (2.9) we can add various boundary conditions. In the case of the first boundary value problem the boundary conditions are

$$\theta = \tilde{\theta}, \; T_\alpha = \tilde{T}_\alpha \text{ on } L, \tag{2.11}$$

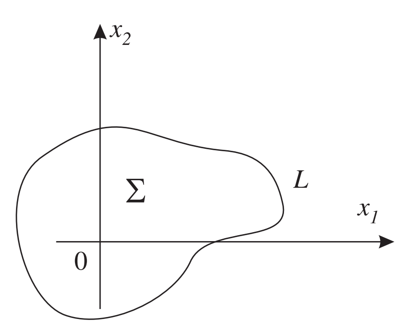

Fig. 1. The cross-section of the cylinder.