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Comment on “A novel analytical solution of mixed convection about an inclined flat plate embedded in a porous medium using the DTM-Padé” by M. M. Rashidi, N. Laraqi and S. M. Sadri, Int. J. Thermal Sciences, 49 (2010) 2405–2412

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ABSTRACT

It is shown that one of the basic differential equations, as well as other important equations of the paper quoted in the title are erroneous and thus the reported results are invalid. This “accident” may have happened because the authors have taken their basic equations from the book of Nield and Bejan (2006) which had printing errors.

In a recent paper of Rashidi et al. [1], the problem of the self-similar mixed convection flows about an inclined flat plate embedded in a saturated porous medium has been revisited. There appears that the authors were not aware of the pioneering paper of Cheng [2] in which this problem has thoroughly been analyzed more than three decades years ago. The same holds for a further basic reference in this research field, namely the work of Lai and Kulacki [3] in which the results of Cheng [3] were extended to the case of permeable ($f_w \neq 0$) surfaces with a lateral mass flux (suction and injection) of the fluid, and in which also a comprehensive numerical solution of the problem has been reported. The aim of Rashidi et al. [1] was to (re)solve the pertinent boundary value problem by the so called differential transform method (DTM). The basic equations (1)–(11), as well as the corresponding text of the paper [1] were transcribed nearly verbatim from the Section 8.1.1 of the book of Nield and Bejan [4]. Unfortunately, the authors of [1] have not realized that several equations of the Section 8.1.1 of [4] contain printing errors. Accordingly, the results of [1] based on these erroneous equations and on further errors committed by the authors, are invalid. For convenience, in the Appendix A the correct versions of these equations are derived in some detail. Specifically, the following equations are concerned.

1. The last Eq. (4) of [1] and the last Eq. (8.4) of [4], both of the same form $U_\infty = Bx^n$, must be replaced by (see the second equation (A.4) of the Appendix)

$$U_\infty = Bx^m \tag{1}$$

   Otherwise, the sentence “The exponent m is related to the angle of inclination $\gamma\pi/2$ (to the incident free stream velocity) by the relation $\gamma = 2m/(m+1)$”, which has been pasted verbatim from [4] in [1], makes no sense.

2. The first Eq. (5) of [1], $\eta = y(U_\infty x/\alpha_m)^{1/2}$, and the first Eq. (8.5) of [4], $\eta = (U_\infty x/\alpha_m)^{1/2}$, both are incorrect and should be replaced by (see Eq. (A.24))

$$\eta = \left(\frac{U_\infty x}{\alpha_m}\right)^{1/2}\frac{y}{x} \tag{2}$$

3. Eq. (6) of [1] and Eq. (8.6) of [4] both have the erroneous form $f_w = -2a/(\alpha_m B)^{1/2}$. The correct expression reads (see Eq. (A.25))

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$$f_w = -\frac{2a}{(m+1)(\alpha_m B)^{1/2}} \tag{3}$$

4. The most serious error with harmful consequences, however, is involved in the basic differential Eq. (8) of [1] and Eq. (8.8) of [4] which both read

$$\theta'' = -\frac{\lambda+1}{2} f\,\theta' + \lambda f \theta (\text{erroneous}) \tag{4}$$

The second term on the right hand side of the above equation must be replaced by $\lambda f'\theta$, i.e., the correct version of Eq. (4) is (see Eq. (A.21))

$$\theta'' = -\frac{\lambda+1}{2} f\theta' + \lambda f'\theta \ (\text{correct}) \tag{5}$$

5. As a consequence, in the basic Eq. (18) of [1] which has been obtained from the above Eq. (4) and which reads

$$(k+2)(k+1)\Theta(k+2) = -\frac{\lambda+1}{2}\sum_{r=0}^{k}(k-r+1)F(r)\Theta(k-r+1) + \lambda\sum_{r=0}^{k}F(r)\Theta(k-r) \tag{6}$$

the second term on the right hand side is also in error. The correct version of this equation which results from the above Eq. (5) is

$$(k+2)(k+1)\Theta(k+2) = -\frac{\lambda+1}{2}\sum_{r=0}^{k}(k-r+1)F(r)\Theta(k-r+1) + \lambda\sum_{r=0}^{k}(r+1)F(r+1)\Theta(k-r) \tag{7}$$

6. Eqs. (12) and (13) of [1],

$$u = \alpha_m\, Pef'(\eta) \tag{8}$$

$$v = -\frac{\alpha_m}{2x} Pe^{1/2}\left[f(\eta) + \eta f'(\eta)\right] \tag{9}$$

specifying the components of the velocity field also contain some errors. The correct forms of these equations are (see Eq. (A.22) and (A.23))

$$\begin{aligned} u &= U_\infty f'(\eta), \\ v &= -\tfrac{\alpha_m}{2x} Pe^{1/2}\left[(\lambda+1)f(\eta) + (\lambda-1)\eta f'(\eta)\right] \end{aligned} \tag{10}$$

where U_∞ is given by the above Eq. (1) with $m = \lambda$, as being required by the similarity reduction of the problem, and $Pe = U_\infty x/\alpha_m$, [1]. Regrettably enough, one also encounters further inconsistencies in [1]. This can be seen in the following way. The second equation which, in addition to the energy equation, connects the dimensionless stream function f to the dimensionless temperature θ is the Darcy-Boussinesq equation. The latter equation results in Eq. (7) of [1], namely

$$f''(\eta) = \pm\frac{Ra}{Pe}\theta'(\eta) \tag{11}$$

with Ra and Pe as being defined by Eqs. (11) of [1] (see also Eqs. (A.18) and (A.20)). Integrating this equation once and bearing in mind the boundary conditions $f'(\infty) = 1$ and $\theta(\infty) = 0$ one obtains

$$f'(\eta) = 1 \pm \frac{Ra}{Pe}\theta(\eta) \tag{12}$$

In particular, for the values of $f''(\eta)$, $\theta'(\eta)$ and $f'(\eta)$ on the plate, $\eta = 0$, the following relationships must hold

$$f''(0) = \pm\frac{Ra}{Pe}\theta'(0) \tag{13}$$

$$f'(0) = 1 \pm \frac{Ra}{Pe} \tag{14}$$

Eqs. (11)–(14) must be satisfied, no matter whether the correct energy equation Eq. (5), or the erroneous one (4) has been applied (although their quantitative content is different in the two cases). Now, let us consider the DTM power series solutions given by Eqs. (20) and (21) of [1], which are valid for the opposing flow case with $Ra/Pe = 0.5$, $\lambda = 1$ and $f_w \equiv f(0) = 1$. From the mentioned equations of [1] one easily obtains $f'(0) = 0.5$, $f''(0) = 0.95096$ and $\theta'(0) = -1.90192$. Although these values are not consistent with the correct energy Eq. (5), all of them are in an excellent agreement with Eqs. (13) and (14). However, while from the Padé approximant of $\theta(\eta)$ given by Eq. (30) of [1] one recovers the above value $\theta'(0) = -1.90192$ of the wall temperature gradient, the Padé approximant of $f(\eta)$ given by Eq. (29) of [1] violates the boundary condition $f(0) = 1$, leading for $f(0)$ to the value 0.5 instead of 1 and for $f'(0)$ to the value $f'(0) = 1.55724 - 1.21246\ 0.5 = 0.95096$ which exceeds the expected value of $f'(0) = 0.5$ by almost 100%. Therefore, Eq. (29) of [1] cannot be correct. Indeed, the exact numerical solution for $Ra/Pe = 0.5$, $\lambda = 1$, $f_w \equiv f(0) = 1$ yields in the opposing case the values $\theta'(0) = -1.71575$ and $f''(0) = 0.857875$ instead of $\theta'(0) = -1.90192$ and $f''(0) = 0.95096$ resulting from the mentioned equations of [1]. It is also worth noticing here that the exact numerical value $\theta'(0) = -1.71575$ lies (quite reasonably) between the values $\theta'(0) = -1.7557$ and $\theta'(0) = -1.6745$ reported in Table 1 of Lai and Kulacki [3] for the opposing flow with $\lambda = 1$, $f_w \equiv f(0) = 1$, and $Ra/Pe = 0.4$ and $Ra/Pe = 0.6$, respectively.

In the same sense, the plots shown in Fig. 12 of [1] also contradict the requirement of the above Eq. (14). Indeed, the values of the mainstream velocity component $u(x,y) = U_\infty f'(\eta)$ on the plate are obtained according to Eq. (14) as

$$u|_{y=0} = U_\infty f'(0) = U_\infty\left(1 \pm \frac{Ra}{Pe}\right) \tag{15}$$

For example, in the case of an aiding flow with $Ra/Pe = 0.5$ and at the distance x where $U_\infty = 2\ m/s$ (as being assumed in the Fig. 12), our Eq. (15) gives $u|_{y=0} = 3\ m/s$, while in Fig. 12 of [1] a value around of $u|_{y=0} = 0.6\ m/s$ can be seen which is five time smaller than the result of Eq. (15). This deviation, which actually originates in the erroneous Eq. (12) of [1] (which is the above Eq. (8)), renders the reliability of the paper once more questionable. It is also worth mentioning here that our Eq. (5) coincides with Eq. (24) and our Eqs. (10) with Eqs. (17) and (18) of Cheng [2] exactly. Obviously, the whole paper of Cheng [3] is free from the errors listed above (see also Eqs. (A.21)–(A.23)).

Returning to the main error of the paper involved in the energy eq. (4) and in its differential transform (6), we would like to illustrate the consequences of this mistake first by an example for which both the erroneous Eq. (4) and the correct Eq. (5) can be solved exactly. Specifically, let us consider the forced convection limiting case of the present model which is obtained for $Ra/Pe \to 0$. In this case the solution of Eq. (12) which satisfies the pertinent boundary conditions is $f(\eta) = f_w + \eta$, [3]. For the sake of simplicity, we further assume that the plate is impermeable, $f_w = 0$, and that the temperature exponent equals unity, $\lambda = 1$ (stagnation flow