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Fire-ball expansion and subsequent shock wave propagation from explosives detonation

Ho-Young Kwak [a,*], Ki-Moon Kang [a], Ilgon Ko [a], Jae-Hoon Kang [b]

[a] Mechanical Engineering Department, Chung-Ang University, Seoul 156 756, Republic of Korea
[b] School of Architecture and Building Science, Chung-Ang University, Seoul 156 756, Republic of Korea

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ABSTRACT

A well-known problem in thermo-hydrodynamics involves the sudden release of the explosion energy, concentrated in a finite volume, to surroundings with uniform density. However, only similarity solutions which have no detailed information on the earlier behavior of the fire-ball are available on this problem. In this study, we obtain a set of analytical solutions for the time dependent radius of an expanding fire-ball after explosives detonation by solving continuity, Euler and energy equations using a "polytrope" assumption for the fire-ball center. Subsequent spherical shock waves that develop from the fire-ball are obtained using the Kirkwood–Bethe hypothesis with Tait's equation of state for water and the ideal gas equation of state for air. The pressure waves emanating from the oscillating bubble in underwater, which has a notably different time scale from the shock wave generation, are obtained using the Rayleigh equation. The calculated results for the period and the maximum radius of bubble that resulted from the fire-ball and the pressure wave from the oscillating bubble in underwater are similar to the observed ones (Swift and Decius, 1946: Yennie and Arons, 1946). The calculated peak pressures as a function of shock radius in air medium were comparable to those observed (Swisdak, 1975).

1. Introduction

With the detonation of a solid explosive, enormous amounts of energy are released and extremely high pressure develops while the solid density remains unchanged. After the detonation process terminates, the high pressure material expands rapidly and accompanied by a strong shock wave, known as ablast wave. Strong explosion accompanying shock waves in a homogeneous media has been thoroughly studied [1]. The distributions of pressure, density and gas velocity with respect to the shock radius were determined to solve the continuity and Euler equations with introducing a similarity variable by Sedov [1], which can also be found in Landau and Lifshitz [2]. The radius and velocity of the shock wave front were obtained by dimensional analysis [3]. However, the similarity solutions obtained in previous works do not adequately cover the region very near to the explosion source, i.e., they provide the velocity of the fire-ball only later time after the explosion.

Recently, numerical studies analyzing strong blast wave have been performed by several authors [4,5]. Especially, numerical simulation of underwater explosion was tried by various methods such as arbitrary Lagrangian–Eulerian formulation [6], indirect boundary element method [7], time-integration boundary-integral method [8] and adaptive solution technique [9]. In their study, the surrounding fluid was treated as either incompressible [4,7,8] or compressible [4,6,9] medium. However, the gas behavior inside the bubble was obtained by the relation of PV^n = constant. As is well known, the index n can range in the interval from 1 (isothermal) to the ratio of the specific heats γ (adiabatic) in polytropic approximation which assumes the uniform pressure and temperature for the gas inside the bubble intrinsically. Furthermore, the earlier stage of the expansion process which provides crucial information to obtain the subsequent shock propagation has never been treated properly by the numerical simulation [4,5]. In fact, initial radius and the corresponding pressure for the motion of the bubble evolved from the detonation products were chosen to match the observed bubble period [4]. On the other hand, with R_o and P_o which were taken as the initial conditions for the expansion of detonation products, the Rayleigh equation yields extremely large bubble period and the maximum radius, and consequently considerably large value in the shock strength.

The motion of the bubble evolved from the detonation products underwater is characterized by two different time scales having two orders of magnitude between the expansion of the detonation

* Corresponding author.
E-mail address: kwakhy@cau.ac.kr (H.-Y. Kwak).

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products and the pulsating of bubble [10]. It is really hard task to treat the underwater explosions involving several complex physical processes.

In this study, we obtained a set of analytical solutions for the time dependent radius of an expanding fire-ball after detonation of explosives by solving for the continuity, Euler and energy equations with a "polytrope" assumption at the fire-ball center. Spherical shock waves that developed from the rapidly expanding fire-ball were obtained using the Kirkwood–Bethe hypothesis [11] with Tait's equation of state for water and with the ideal gas equation of state for air medium. Subsequent pressure waves emanating from the oscillating bubble in underwater are obtained using the Rayleigh equation [12]. In an air environment, the Rayleigh equation, neglecting the pressure term, was used to calculate the behavior of the fire-ball after rapid expansion.

2. Continuity and the Euler equation and their solutions

The equation for continuity and the Euler equation for an irrotational fluid are given as

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\left(\rho\vec{u}\right) = 0 \tag{1}$$

$$\frac{\partial\vec{u}}{\partial t}+\vec{u}\cdot\nabla\vec{u} = -\frac{1}{\rho}\nabla P \tag{2}$$

Here $\vec{u}$ is the fluid velocity. With decomposition of the gas density in spherical symmetry as $\rho = \rho_o(t)+\rho_r(r,t)$ the continuity equation becomes

$$\left[\frac{\partial\rho_o}{\partial t}+\rho_o\nabla\cdot\vec{u}\right]+\frac{D\rho_r}{Dt}+\rho_r\nabla\cdot\vec{u} = 0 \tag{3}$$

where the notation of the total derivative used here is $D/D = \partial/\partial t+u\partial/\partial r$.

The rate of change of the density of a material particle can be represented by the rate of volume expansion of that particle in the limit [13] $V\to 0$, or

$$\nabla\cdot\vec{u} = \lim_{V\to o}\left(\dot{V}/V\right) = 3\dot{R}/R \tag{4}$$

With this result, one can have radial dependent velocity profile inside the fire-ball from the continuity equation.

$$u = \dot{R}r/R \tag{5}$$

This linear velocity means that the fire-ball expands homologously [14]. In other words, every mass point during expansion may be traced back to a single point, the center of the fire-ball. With this velocity profile, one can obtain the following quadratic density profile. The fully detailed derivation can be found in Kwak and Yang [15] and Kwak and Jun [16].

$$\rho(r,t) = \frac{b}{R^3(t)}+\frac{ar^2}{R^5(t)} \tag{6}$$

where "a" and "b" are constants related to the mass of the fire-ball.

Substituting Eqs. (5) and (6) into the Euler equation given in Eq. (2), we have the following pressure profile inside the fire-ball:

$$P = P_0(t)-\left(\frac{b}{2}\frac{r^2}{R^2}+\frac{a}{4}\frac{r^4}{R^4}\right)\frac{\ddot{R}}{R^2} \tag{7}$$

At the center of the fire-ball, the above equation with a boundary condition, P = 0 at r = R becomes:

$$P_0 = \left(\frac{a+2b}{4}\right)\frac{\ddot{R}}{R^2} \tag{8}$$

To determine the "a" and "b" in Eq. (6), we need an equation of state for the gas inside the fire-ball. Excluding the viscous dissipation term, the energy equation with internal and enthalpy representation can be written as [15,17]

$$\rho C_v\frac{DT}{Dt} = -P\nabla\cdot\vec{u}-\nabla\cdot\vec{q} \tag{9}$$

$$\rho C_p\frac{DT}{Dt} = \frac{DP}{Dt}-\nabla\cdot\vec{q} \tag{10}$$

where C_v and C_p are the specific heats at constant-volume and at constant-pressure, respectively, and $\vec{q}$ is the heat flux. Eliminating $\nabla\cdot\vec{q}$ from Eqs. (9) and (10) give us

$$\rho\left(C_p-C_v\right)\frac{DT}{Dt} = \rho R_g\frac{DT}{Dt} = P\nabla\cdot\vec{u}+\frac{DP}{Dt} \tag{11}$$

where R_g is the gas constant. At the center of the fire-ball, the above equation reduces to

$$\rho_0 R_g\frac{\partial T_0}{\partial t} = 3P_0\frac{\dot{R}}{R}+\frac{\partial P_0}{\partial t} \tag{12}$$

which yields the following ideal gas relation at the center of the fire-ball.

$$P_oR^3/T_o = const. \tag{13}$$

where P_o and T_o are the gas pressure and temperature, respectively, at the fire-ball center.

The adiabacity condition at the fire-ball center can be written as:

$$P_0 = \kappa\rho_0^{\gamma_T} \tag{14}$$

where γ_T is the specific heat ratio for the detonation products of the explosives and κ is some constant.

The assumption of the polytrope at the center is equivalent to the assumption that the center is neither a heat source nor a heat sink, i.e., $\nabla\cdot\vec{q} = 0$ at the center. When the velocity, density and pressure profiles are obtained, and are given in Eqs. (5–7), we can solve the energy equation with the boundary condition given in Eq. (14) to obtain the following results.

$$a = -b = -15M/8\pi \tag{15}$$

$$\rho(r,t) = \frac{b}{R^3(t)}\left[1-\left(\frac{r}{R}\right)^2\right] \tag{16}$$

$$T(r,t) = T_s+\frac{R\ddot{R}}{4R_g}\left[1-\left(\frac{r}{R}\right)^2\right] \tag{17}$$

$$P(r,t) = \frac{b}{4}\frac{\ddot{R}}{R^2}\left[1-\left(\frac{r}{R}\right)^2\right]^2 \tag{18}$$

where M is the mass of the explosive.

Note that even though the temperature at the surface of the fire-ball T_s is included in Eq. (17), it cannot be calculated using the ideal gas law because the gas pressure at the surface of the fire-ball becomes null from Eq. (18), which indicates that the properties at the surface of the fire-ball cannot be defined. In fact, the pressure, density and gas velocity distributions behind the shock front may be obtained from Sedov analysis [1].