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Performance analysis of open and ducted wind turbines

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HIGHLIGHTS

- A new semi-analytical actuator disk model is applied to open and ducted wind turbines.
- Open and ducted turbines are compared for the same rotor load.
- Results show that a properly ducted wind turbine can extract a higher power.
- The proposed model is compared with a CFD based method.
- The analysis code is freely available contacting the authors.

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ABSTRACT

In this paper the analysis of the aerodynamic performance of ducted wind turbines is carried out by means of a nonlinear and semi-analytical actuator disk model. It returns the exact solution in an implicit formulation as superposition of ring vortices properly arranged along the duct surface and the wake region. In comparison with similar and previously developed models, the method can deal with ducts of general shape, wake rotation and rotors characterised by radially varying load distributions. Moreover, the nonlinear mutual interaction between the duct and the turbine, and the divergence of the slipstream, which is particularly relevant for heavily loaded rotors, are naturally accounted for. Present results clearly show that a properly ducted wind turbine can swallow a higher mass flow rate than an open turbine with the same rotor load. Consequently, the ducted turbine achieves a higher value of the extracted power. The paper also presents a detailed comparison between the aforementioned nonlinear and semi-analytical actuator disk method and the widely diffused CFD actuator disk method. The latter is based on the introduction of an actuator disk model in a CFD package describing the effects of the rotor through radial profiles of blade forces distributed over a disk surface. A set of reference numerical data, providing the inviscid axisymmetric velocity and pressure field distributions, are generated with controlled accuracy. Owing to an in-depth analysis of the error generated by the semi-analytical method and to the exactness of the solution in its implicit form, the collected data are well-suited for code-to-code validation of existing or newly developed computational methods.

1. Introduction

Although the linearised actuator disk of Rankine [1] and Froude [2] represents the oldest analytical performance analysis method for propellers and turbines, it still constitutes a widely diffused and employed design/analysis tool, especially in the wind turbine field.

In 1962 Wu [3] introduced the first nonlinear actuator disk model and, although he did not carry out any numerical calculations, the theoretical basis of his approach still represent a reference in this specific field. Greenberg and Powers [4], Greenberg [5], and Conway [6,7] proposed significant improvements in the analytical and numerical treatment to the theory of Wu [3]. Recently Bontempo and Manna [8] and Bontempo et al. [9–11] have presented an extension to ducted rotors (see Fig. 1) of the actuator disk of Conway [7]. The exact solution furnished in [8] results from the superposition of ring vortices properly arranged along the duct surface and the wake region. The method can deal with ducts of general shape, wake rotation and rotors characterised by non-uniform, heavy load distributions. Moreover, it can naturally account for the shape of the slipstream and the nonlinear mutual interaction between the duct and the rotor. Finally, also because of its cost effectiveness, it is well-suited for the design and/or analysis of ducted turbines.

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Nomenclature

a_m	load polynomial coefficients
A_n	tangential vorticity polynomial coefficients
$\hat{b} = \sigma_{ad}^2 a_1/U_\infty \Psi_{\sigma_{ad}}$	dimensionless load magnitude parameter
C	contour of the duct section
$c,\ \tilde{c}$	curvilinear abscissa along the duct section
C_P	power coefficient
$C_{p,w} = \frac{p-p_\infty}{1/2\rho U_\infty^2}$	wall pressure coefficient
$C_T,\ C_{T,rot},\ C_{T,duct}$	total, rotor and duct thrust coefficients
D	rotor diameter
H	Bernoulli constant
H_∞	Bernoulli constant at upstream infinity
$\mathcal{H} = \Delta H_{\text{across the disk}}$	rotor load
$\hat{\mathcal{H}} = 2\mathcal{H}/U_\infty^2$	load coefficient
J_n	Bessel function of the first kind and order n
$k(c,\tilde{c}),\ K(c_m,c_n)$	coupling coefficients
$\dot{m}$	mass flow
M_p	number of panels on the duct section
n_z	number of axial stations
$n_{zs} \times n_{rs}$	number of axial and radial stations
P	power extracted by the turbine
p	static pressure
$Q_{1/2}$	Legendre function of the second kind and degree 1/2
T	thrust experienced by the device
$\boldsymbol{u}$	velocity vector
$u,\ v,\ w$	axial, radial and tangential components of the velocity
$u',\ v'$	axial and radial components of the velocity induced by a ring vortex
U_∞	free stream velocity
$u_{ad},\ v_{ad}$	axial and radial components of the velocity induced by the actuator disk wake ring vortex distribution
$u_{ad,m},\ v_{ad,m}$	axial and radial components of the velocity induced by the actuator disk wake ring vortex distribution at the m-th panel
W	work extracted by the turbine
$z,\ r$	auxiliary axial and radial coordinates

Greek symbols

α	angle between the chord and the axial direction
$\beta(c)$	local duct profile slope
γ_{ad}	strength density of the actuator disk ring vortex distribution
γ_d	strength density of the duct ring vortex distribution
Δc_n	length of the n-th panel
Δp_{ad}	static pressure jump across the disk
ϵ	tip gap
$\zeta,\ \sigma,\ \theta$	axial, radial and tangential coordinates
ζ_{le}	leading edge axial position
$\zeta_m,\ \sigma_m$	axial and radial coordinate of the control point of the m-th panel
κ	single ring vortex strength
$\lambda = \Omega\sigma_{ad}/U_\infty$	tip speed ratio
ρ	density
σ_{ad}	actuator disk radius
σ_{le}	leading edge radial position
σ_s	slipstream edge
$\boldsymbol{\Psi}$	Stokes stream function
Ψ'	Stokes stream function induced by a ring vortex
Ψ_0	Stokes stream function on the duct
Ψ_{ad}	Stokes stream function evaluated at $(\zeta = 0, \sigma = \sigma_{ad})$
Ψ_d	Stokes stream function induced by the duct ring vortex distribution and by the free stream
$\boldsymbol{\omega} = (\omega_\zeta, \omega_\sigma, \omega_\theta)$	vorticity vector
Ω	turbine angular velocity

Acronyms

CFDDT	CFD actuator disk method for a ducted turbine
CFDOT	CFD actuator disk method for a open turbine
SADT	semi-analytical actuator disk method for a ducted turbine
SAOT	semi-analytical actuator disk method for a open turbine

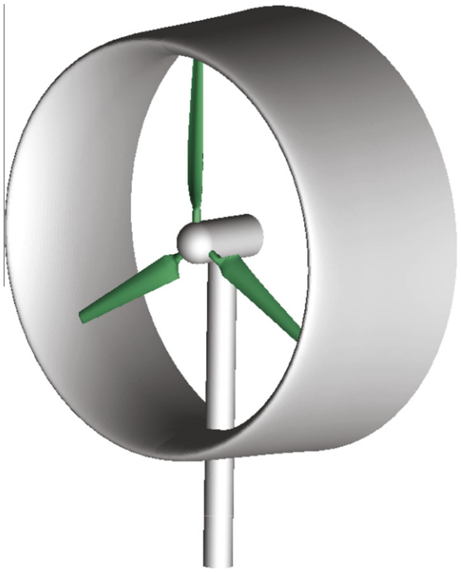

Fig. 1. Ducted wind turbine: a typical layout.

Recent developments in the field of wind turbine wake modelling are directed towards the combined use of actuator disk and CFD techniques (here and after referred to as "CFD actuator disk"). The first applications of the numerical actuator disk are due to Sørensen and Myken [12] and Sørensen and Kock [13]. In these papers the unsteady and axisymmetric Navier–Stokes equations are solved outside the rotor plane, while the actuator disk is modelled through volume forces evaluated with the help of tabulated airfoil data. Madsen [14] concentrated on the accuracy of the blade element method (see for example Lanzafame and Messina [15]); the accuracy assessment was carried out comparing the results of the blade element method with those obtained solving the Navier–Stokes equations for the flow through an actuator disk. As a preliminary step in the development of a computational method capable to analyse the aerodynamic of wind turbine farms, Masson et al. [16] predicted the steady, incompressible and axisymmetric flow around an isolated wind turbine by solving the Navier–Stokes equations. Sørensen et al. [17] combined the actuator disk model with the Navier–Stokes equations to analyse various wake states like the turbulent wake and the vortex ring state. Further numerical improvements are due to Réthoré and Sørensen [18]. More recent developments of the method refer to the so-called 3D actuator line or actuator surface model [19–28].

The CFD actuator disk method is also employed to study several more complex flow cases like yawed rotors [29], coned rotors