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Mixed convection flow near the axisymmetric stagnation point on a stretching or shrinking cylinder

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A B S T R A C T

The steady mixed convection flow near an axisymmetric stagnation point on a stretching or shrinking vertical cylinder is considered. The equations for the fluid flow and temperature fields reduce to similarity form that involves a Reynolds number R, a mixed convection parameter λ, a parameter γ representing the motion of the cylinder and the Prandtl number σ. Numerical solutions of the similarity equations are obtained for representative values of these parameters, which show the existence of critical values $\lambda_c = \lambda_c\,(R,\gamma,\sigma)$ with the existence of dual solutions in the opposing ($\lambda < 0$) case. The variation of the skin friction coefficient $f''(1)$ and heat flux $\theta'(1)$ with λ and R when $\sigma = 1.0$ are shown graphically. Also variations of λ_c with R, and γ are determined again in the case when $\sigma = 1.0$. It is found that in the aiding flow ($\lambda > 0$) case solutions are possible for all λ and the asymptotic solution in the limit $\lambda \to \infty$ is obtained. The nature of the solution in the asymptotic limit of large values of R is also treated in the cases when λ is of $O\,(1)$ and when λ is of $O\,(R)$.

1. Introduction

The dynamics of fluid flow due to continuous moving surfaces is important in many practical applications, for example in the extrusion of plastic sheets, paper production, glass blowing, metal spinning, drawing plastic films, see, for example, Altan et al. [1], Fisher [2], Tadmor and Klein [3] and Fang et al. [4]. The quality of the final product depends on the rate of heat and mass transfer between the moving surface and the flow. The laminar boundary-layer flow driven by continuously moving solid surfaces has already received considerable attention since the pioneering investigation by Sakiadis [5,6], partly because of its practical relevance in various extrusion processes but also because of its role as a canonical flow problem in the boundary-layer theory of Newtonian and non-Newtonian fluid mechanics.

Laminar free and mixed convection boundary-layer flow along a static vertical cylinder has been studied by many investigators, see, for example, Nagendra et al. [7], Chen and Mucoglu [8], Mucoglu and Chen [9], Lee et al. [10], Mahmood and Merkin [11]. However, there are only a relatively few studies on the boundary-layer flow past a moving cylinder, Karnis and Pechoc [12], Lin and Shih [13,14], Takhar et al. [15]. Riley [16] in a very interesting paper studied the free convection boundary-layer flow on a heated slender circular cylinder which moves with a constant velocity in the direction of gravity as it emerges from an orifice with an excess temperature T_0 above that of the ambient temperature T_∞ and disappears into another orifice with an excess temperature $T_1 < T_0$. Riley [16] has shown that, contrary to the corresponding problem of the cooling of a two-dimensional vertical thin sheet considered by Kuiken [17], this problem does not admit a similarity solution. Thus Riley solved the governing partial differential equations using a series solution in terms of $(x_0 - x)$, where x is the coordinate measuring distance along the slender circular cylinder and x_0 is the singular point in Kuiken's [17] similarity solution. Besides, Wang [18] has studied the steady flow and heat transfer characteristics of impermeable stretching cylinder and then this problem has been extended to the case of permeable stretching cylinder by Ishak et al. [19]. Recently, Wang [20] has studied the problem of free convection flow over a vertical stretching cylinder using the similarity transformation. He has also published a review paper of similarity solutions of viscous fluid due to stretching and shrinking boundaries (Wang [21]).

Stagnation point flows appear in virtually all flow fields of engineering and scientific interest. In some situations the flow is stagnated by a solid wall, while in others cases a free stagnation

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Nomenclature

a	radius of the cylinder, m
b	stretch or shrink rate, s^{-1}
$f(\eta)$	dimensionless stream function, similarity variable
$f''(1)$	dimensionless skin friction coefficient
g	acceleration due to gravity, m s^{-2}
r	radial coordinate, m
R	Reynolds number
T_∞	ambient temperature, K
T_w	temperature of the cylinder surface, K
u, w	velocity components along r and z axes, m s^{-1}
U_∞	free stream velocity, m s^{-1}
w_0	velocity of the moving cylinder, m s^{-1}
z	axial coordinate, m

Greek symbols

β	volume expansion coefficient, K^{-1}
ΔT	characteristic temperature difference, K
γ	ratio of cylinder stretch or shrink rate to the stagnation point strain rate
η	dimensionless similarity independent variable
λ	dimensionless mixed convection parameter
λ_c	critical value of λ
μ	large R, λ solution, $\lambda = \mu R$
ν	kinematic viscosity, m^2 s^{-1}
$\theta(\eta)$	dimensionless temperature
$\theta'(1)$	dimensionless heat flux
σ	Prandtl number
ψ	stream function, m^3 s^{-1}

point or line exists interior to the fluid domain. Stagnation flows may be characterized as inviscid or viscous, steady or unsteady, two-dimensional or three-dimensional, symmetric or asymmetric, normal or oblique, homogeneous or having two immiscible fluids and forward or reverse, Weidman and Putkaradze [22]. It seems that Wang [23] was the first to have investigated the stagnation flow normally directed to the surface of a circular cylinder, and since then a number of variations that take into account unsteady flow effects, cylinder translation and rotation, and wall transpiration have been published, Gorla [24,25], Cunning et al. [26], Takhar et al. [27]. The problem of mixed convection in an axisymmetric stagnation flow on a static vertical cylinder corresponding to an isothermal cylinder ($n = 0$) and a linearly increasing temperature ($n = 1$) respectively has been studied numerically by Gorla [28,29] for different values of the Prandtl number by considering both the assisting and opposing flow situations. This problem has been further extended by Revnic et al. [30]. Numerical solutions are obtained in Ref. [30] for representative values of the Reynolds number R, the mixed convection parameter λ and the Prandtl number σ, which show the existence of a critical value λ_c of λ (dependent on both R and σ) for the existence of solutions in the opposing flow ($\lambda < 0$) case. In is also shown that in the aiding flow ($\lambda > 0$) case solutions are possible for all values of λ and the asymptotic limit $\lambda \to \infty$ is obtained. The limits of large and small R are also treated in Ref. [30] and the nature of the solution in the asymptotic limit of large Prandtl number is discussed.

The aim of the present paper is to extend the problem considered by Gorla [29] and Revnic et al. [30], namely to consider the steady axisymmetric stagnation point flow of a viscous fluid on a stretching or shrinking vertical cylinder. It is worth mentioning here that the study on shrinking sheet near the stagnation point was first published by Wang [31] who found that solutions do not exist for large shrinking rates and may be non-unique in the two-dimensional case. Ishak et al. [32] obtained the similarity solutions of stagnation point flow over a shrinking sheet immersed in an incompressible micropolar fluid. Recently, Lok et al. [33,34] extended Wang's problem to MHD flow for impermeable and permeable shrinking sheet, respectively, where dual solutions exist for small values of magnetic parameter. Very recently, the study on the stagnation flow over a permeable shrinking cylinder in forced convection has been investigated by Lok and Pop [35] and they found that triple solutions exist for some values of shrinking parameter. In this paper, we are mostly concerned with an examination of the effects of a stretching or shrinking vertical cylinder for the specific values of the parameters R, λ and γ, and by considering some asymptotic limits. We start by giving the equations, already derived in Ref. [30], though now with a different boundary condition on the cylinder.

2. Equations

Following directly from Revnic et al. [30] and Gorla [29] we consider the flow of a steady, viscous incompressible fluid at an axisymmetric stagnation point on an impermeable infinite cylinder of radius a stretching or shrinking in a fluid of uniform ambient temperature T_∞. z and r are the cylindrical polar co-ordinates measuring distance in the axial and radial directions respectively. The flow is taken to be axisymmetric about the z-axis and also symmetric about the $z = 0$ plane, with the stagnation line being at $z = 0$, $r = a$. We assume that the cylinder moves with the velocity $w_0(z) = 2bz$ along its length where b is a constant that can be either positive (stretching cylinder) or negative (shrinking cylinder), and that the temperature of the cylinder is, from Ref. [30], $T_w = T_\infty + \Delta T\, z/a$ and the outer flow is, from Ref. [29], $u = -(U_\infty/a)(r - a^2/r)$, $w = 2U_\infty(z/a)$ where u and w are the velocity components in the r and z directions. ΔT is a constant which can be either positive (aiding flows) or negative (opposing flows).

Then, again following directly from Refs. [29,30], we can reduce the problem to the similarity form, namely to

$$\eta f''' + f'' + R(ff'' + 1 - f'^2) + \lambda\theta = 0$$
$$\eta\theta'' + \theta' + \sigma R(f\theta' - f'\theta) = 0 \tag{1}$$

on $1 \le \eta < \infty$, where primes denote differentiation with respect to the independent variable $\eta = (r/a)^2$, subject to the boundary conditions

$$f(1) = 0,\ \ f'(1) = \gamma,\ \ \theta(1) = 1,\ \ f' \to 1,\ \ \theta \to 0 \ \text{ as } \ \eta \to \infty \tag{2}$$

where we have taken the stream function ψ as $\psi = U_\infty a z f(\eta)$ with U_∞ being the free stream velocity. Here R is a Reynolds number, λ is a mixed convection parameter and γ is a parameter, which can be positive or negative, measuring the longitudinal velocity of the cylinder defined by

$$R = \frac{U_\infty a}{2\nu},\quad \lambda = \frac{g\beta a^2 \Delta T}{8U_\infty \nu},\quad \gamma = \frac{ab}{U_\infty} \tag{3}$$

and σ is the Prandtl number. These are essentially the same equations treated in some detail in Ref. [30] and for a specific value of R in Ref. [29], apart from the condition in Eq. (2) that $f'(1) = \gamma$. In both Refs. [30] and [29] it was assumed that $\gamma = 0$.

Our aim here is to examine the effects that having $\gamma \neq 0$ has on the results reported in Ref. [30] through obtaining numerical