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A mathematical model for swallowing of food bolus through the oesophagus under the influence of heat transfer

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A R T I C L E   I N F O

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A B S T R A C T

A mathematical model is constructed to study the influence of heat transfer in swallowing of food bolus (water) through the oesophagus. The food bolus is supposed to be viscous fluid and the geometry of wall surface of oesophagus is considered as peristaltic wave. The expressions for temperature field, axial velocity, transverse velocity, volume flow rate, pressure gradient, local wall shear stress, mechanical efficiency, stream function and reflux limit are obtained under the assumptions of long wavelength and low Reynolds number. The effect of heat transfer on two inherent phenomena (reflux and trapping) of peristaltic flow is discussed numerically. The comparative study of integral and non-integral number of wave propagating along the channel is discussed under influence of emerging physical parameters. Revelation is that when the magnitude of Grashof number and thermal conductivity increase the pressure along the entire length of the channel reduces whereas the efficiency of pumping increases. Reflux region is found to be increasing function of the both parameters. It is found that the size of upper trapped bolus contracts while size of lower trapped bolus expands with increasing the effect of heat transfer.

1. Introduction

Bio-fluids are propelled from one place to another place by continuous process of muscle contraction and relaxation is called peristaltic transport. An interesting fact is that in the oesophagus where the movement of masticated food is due to peristalsis, food moves from the mouth to stomach even when upside down. The flow of urine in ureters, the vasomotion of blood, food mixing, chyme movement in intestinal, transport of bile in bile ducts and transport of spermatozoa in cervical canal are some important physiological examples where peristalsis is prevalent. The periodic lateral movements of these vessels are due to electro-chemical reaction taking place in the body. Although peristalsis is well known mechanism for biologists, investigation of it from mechanical point of view began very late. Latham [1] ignited the investigation by analytical and experimental approaches in his MS thesis. Shapiro et al. [2] theoretically examined the peristaltic flow of viscous fluid induced by sinusoidal wall propagation. They performed the analysis under long wavelength and low Reynolds number assumptions and discussed the phenomena of reflux and trapping during peristalsis. Later several workers, besides these pioneers, enriched the knowledge and information on this topic with valuable contributions.

Heat transfer is the passage of thermal energy from a hot body to a colder body. When a physical body, e.g., an object or fluid, is at different temperature than it's surrounding or another body, transfer of thermal energy, also known as heat transfer or heat exchange. In physiology, it is used to study the properties of tissues. Recent advances in the application of heat (hyperthermia), radiation (laser therapy), and coldness (cryosurgery), as means to destroy undesirable tissues, such as cancer, have stimulated much interest in the study of modelling in tissue. Radiofrequency therapy is useful to treat more diseases such as tissue coagulation, the primary liver cancer, the lung cancer and the reflux of stomach acid. Some authors [3–18] have reported the influence of heat transfer on peristaltic flow of Newtonian and non-Newtonian fluids (Reiner Rivlin fluid, Jeffrey-six constant fluid, second grade fluid, third order fluid, fourth grade fluid, Herschel Bulkley fluid and Walter's B fluid) with or without the effect of magnetic field through uniform/non-uniform/asymmetric channels/vertical annulus and porous medium. They discussed the characteristics of physical parameters on flow behaviour.

Oesophagus is a collapsible 18–26 cm long and 1.5–2.5 cm wide muscular tube stretching from the upper sphincter to the lower sphincter. The upper sphincter is an inlet and it regulates the passage of the masticated food stuff from the mouth to the

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oesophagus, while the lower sphincter acts as an outlet and controls the passage of the fluid to the stomach for further treatment in the digestive system. The oesophagus gets activated to full distension, only when a bolus knocks at the upper sphincter and a message is transmitted to the oesophagus; otherwise it remains collapsed. The swallowing of the food bolus takes place due to periodic contraction of the oesophageal wall. Pressure due to reflexive contraction is exerted on the posterior part of the bolus and the anterior portion experiences relaxation so that the bolus moves ahead. The contraction is practically not symmetric; yet it contracts to zero lumen and squeezes it marvellously without letting any part of the food bolus slip back in the opposite direction. Any imbalance may result into a retrograde motion.

Most of studies of peristaltic transport of Newtonian and non-Newtonian fluids have considered periodic peristaltic waves in infinite tubes, ignoring the inherently non-steady effects associated with the finite length tubes encountered in real peristaltic pumps. Li and Brasseur [19] have theoretically worked on it for Newtonian fluids considering finite length tubes, and focused the study on both local and global dynamics. The issue of local dynamics such as spatial-temporal variations in local stresses in terms of the motility and efficacy of the transport process was raised by Brasseur and Dodds [20]. They found close resemblance with the experimental findings of Dodds [21]. Misra and Pandey [22] who investigated similar phenomena for power-law fluids came to similar conclusions, although they didn't discuss the experimental findings. Pandey and Tripathi [23–26] have recently worked for MHD, Maxwell, Jeffrey and Casson fluids respectively. Some applications of fractional calculus in peristaltic flow of visco-elastic fluids have been recently presented in Refs. [27–30].

In the fluid mechanics, it is found that temperature plays an important role during the fluid flows. Considering this fact, we design a mathematical model to study the influence of heat transfer on swallowing of food bolus through the oesophagus. For this mathematical model, peristaltic transport of viscous fluid under effect of heat transfer in finite length channel is considered. From engineering point of view, this model may also be applied to study the water flow through a mechanical peristaltic pump. The case of propagation of a non-integral number of waves along the channel walls is studied, which is inherent characteristics of finite length vessels.

2. Mathematical model

The constitutive equation of wall geometry (cf. Fig. 1) due to propagation of train waves is given as

$$\tilde{h}\left(\tilde{\xi},\tilde{t}\right) = a-\tilde{\phi}\cos^2\frac{\pi}{\lambda}\left(\tilde{\xi}-c\tilde{t}\right) \tag{1}$$

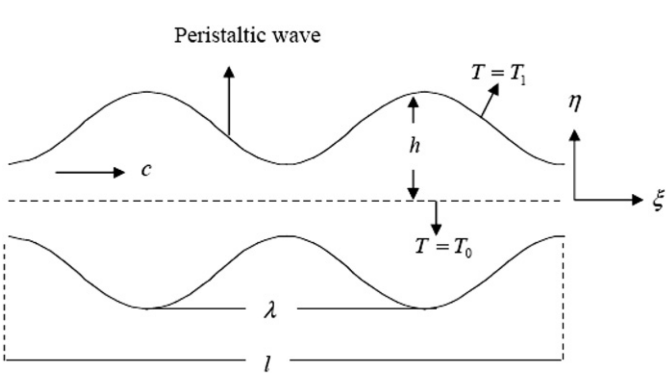

Fig. 1. Geometry of the problem.

where $\tilde{h},\tilde{\xi},\tilde{t},a,\tilde{\phi},\lambda$ and c represent transverse vibration of the wall, axial coordinate, time, half width of the channel, amplitude of the wave, wavelength and wave velocity respectively.

The equations governing the motion of incompressible viscous fluid with heat transfer are given as

$$\left.\begin{aligned}
&\rho\left(\frac{\partial}{\partial\tilde{t}}+\tilde{u}\frac{\partial}{\partial\tilde{\xi}}+\tilde{v}\frac{\partial}{\partial\tilde{\eta}}\right)\tilde{u} = -\frac{\partial\tilde{p}}{\partial\tilde{\xi}}+\mu\left(\frac{\partial^2\tilde{u}}{\partial\tilde{\xi}^2}+\frac{\partial^2\tilde{u}}{\partial\tilde{\eta}^2}\right)+\rho g\alpha(T-T_0),\\
&\rho\left(\frac{\partial}{\partial\tilde{t}}+\tilde{u}\frac{\partial}{\partial\tilde{\xi}}+\tilde{v}\frac{\partial}{\partial\tilde{\eta}}\right)\tilde{v} = -\frac{\partial\tilde{p}}{\partial\eta}+\mu\left(\frac{\partial^2\tilde{v}}{\partial\tilde{\xi}^2}+\frac{\partial^2\tilde{v}}{\partial\tilde{\eta}^2}\right),\\
&\frac{\partial\tilde{u}}{\partial\tilde{\xi}}+\frac{\partial\tilde{v}}{\partial\tilde{\eta}} = 0,\\
&\rho c_p\left(\frac{\partial}{\partial\tilde{t}}+\tilde{u}\frac{\partial}{\partial\tilde{\xi}}+\tilde{v}\frac{\partial}{\partial\tilde{\eta}}\right)T = k\left(\frac{\partial^2 T}{\partial\tilde{\xi}^2}+\frac{\partial^2 T}{\partial\tilde{\eta}^2}\right)+\Phi
\end{aligned}\right\}, \tag{2}$$

where $\rho,\tilde{u},\tilde{v},\tilde{\eta},\tilde{p},\mu,g,\alpha,T,c_p,k$ and Φ denote the fluid density, axial velocity, transverse velocity, transverse coordinate, pressure, fluid viscosity, acceleration due to gravity, coefficient of linear thermal expansion of fluid, temperature, specific heat at constant pressure, thermal conductivity and constant heat addition/absorption.

The temperatures at the centre line and the wall of the peristaltic channel are given as

$$T = T_0 \text{ at } \eta = 0, T = T_1 \text{ at } \eta = h, \tag{3}$$

where T_0 is the temperature at centre line and T_1 is the temperature on the wall of peristaltic channel.

We then introduce the following non-dimensional parameters

$$\left.\begin{aligned}
&\xi = \frac{\tilde{\xi}}{\lambda},\eta = \frac{\tilde{\eta}}{a},t = \frac{c\tilde{t}}{\lambda},u = \frac{\tilde{u}}{c},v = \frac{\tilde{v}}{c\delta},\delta = \frac{a}{\lambda},h = \frac{\tilde{h}}{a},l = \frac{\tilde{l}}{\lambda},\phi = \frac{\tilde{\phi}}{a},\\
&p = \frac{\tilde{p}a^2}{\mu c\lambda},\psi = \frac{\tilde{\psi}}{ac},Q = \frac{\tilde{Q}}{ac},\mathrm{Re} = \frac{\rho ca\delta}{\mu},\mathrm{Gr} = \frac{g\rho^2\alpha a^3(T_1-T_0)}{\mu^2}\\
&\theta = \frac{T-T_0}{T_1-T_0},\beta = \frac{a^2\Phi}{k(T_1-T_0)},P_r = \frac{\mu c_p}{k}.
\end{aligned}\right\} \tag{4}$$

where $\delta,\tilde{l},\tilde{\psi},\tilde{Q},\mathrm{Re},\mathrm{Gr},\theta,\beta$ and P_r are wave number, length of the channel, stream function, volume flow rate, Reynolds number, Grashof number, dimensionless temperature, dimensionless heat source/sink parameter and Prandtl number. Using the low Reynolds number and large wavelength approximations, Eqs. (1–3) reduce to

$$h(\xi,t) = 1-\phi\cos^2\pi(\xi-t), \tag{5}$$

$$\left.\begin{aligned}
&\frac{\partial p}{\partial\xi} = \frac{\partial^2 u}{\partial\eta^2}+\mathrm{Gr}\theta,\\
&\frac{\partial p}{\partial\eta} = 0,\\
&\frac{\partial u}{\partial\xi}+\frac{\partial v}{\partial\eta} = 0,\\
&\frac{\partial^2\theta}{\partial\eta^2}+\beta = 0,
\end{aligned}\right\} \tag{6}$$

$$\theta = 0 \text{ at } \eta = 0, \theta = 1 \text{ at } \eta = h. \tag{7}$$

The following boundary conditions are imposed on the governing equations to model the problem under consideration:

$$\left.\frac{\partial u}{\partial\eta}\right|_{\eta=0} = 0, \text{ i.e., the regularity condition}, \tag{8}$$