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On the relation between Nusselt and Péclet number in high Péclet number convective heat transfer

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ABSTRACT

For the steady forced convection cooling of an isothermal spherical object in an axially symmetric incompressible fluid flow at high Péclet numbers, where the heat transport occurs mainly in a thin thermal boundary layer near the surface of the object, the relation: $Nu \propto Pe^{1/2}$ or $Nu \propto Pe^{1/3}$ between the Nusselt number and the Péclet number is generally found. In this paper we investigate for the general case of a 3-dimensional object immersed in a 3-d incompressible fluid flow the connection between these two characteristic numbers and obtain a similar relation: $Nu \propto Pe^{1/(k+2)}$ with integer values $k \geq 0$, where k depends only on the boundary conditions of the fluid flow field at the surface of the object.

1. Introduction

This work was triggered by the search for a physical model, describing the cooling of a liquid, electro-magnetically levitated metal droplet [1] by a poorly heat conducting Argon gas stream. Due to the low thermal conductivity λ_∞ of the gas and the high characteristic speed v_0 of its flow along the droplet of characteristic dimension R_0, the low thermal energy, transferred by heat conduction from the droplet surface in normal direction into the gas, is quickly “blown away” in tangential direction as schematically illustrated in Fig. 1. The heat transferred by conduction from the front-side of a sphere of temperature T_S and of area $4\pi R_0^2/2$ into the fluid, which can roughly be estimated by the left hand side of Eq. (1) (cf. Eq. (5) below)

$$\frac{4\pi R_0^2}{2}\lambda_\infty \frac{T_S}{\delta} \approx 2\pi R_0\delta \, \rho c_p v_0 T_S, \tag{1}$$

is conveyed by convection through a virtual shell of thickness δ and of cross section $\approx 2\pi R_0\delta$ around the sphere. The strength of this convective heat flow can roughly be estimated by the right hand side of Eq. (1) (cf. Eq. (4) below), where ρ denotes the density of the fluid and c_p its specific heat. With the definition of the Péclet number [2] equation (1) finally reads

$$Pe := \frac{v_0 \rho c_p R_0}{\lambda_\infty} \approx \frac{R_0^2}{\delta^2}, \tag{2}$$

showing that for $Pe \gg 1$ the total convective heat transport occurs mainly in a thin thermal boundary layer of typical thickness $\delta \ll R_0$ around the droplet.

The steady heat transfer from an isothermal sphere of radius R_0 into an incompressible fluid stream at high Péclet number has been the subject of different theoretical investigations since many years [3–6]. It turns out in these papers, that the Nusselt number

$$Nu := \frac{2P}{4\pi R_0 \lambda_\infty (T_S - T_\infty)}, \tag{3}$$

which represents a dimensionless form of the convective heat (energy) flow P between the isothermal sphere of temperature T_S and a heat sink of temperature T_∞ at infinity, is only proportional to $Nu \propto Pe^{1/3}$, see Ref. [3], or to $Nu \propto Pe^{1/2}$, see Refs. [4–6], dependent on the particular velocity field (Stokes flow, constant flow field, potential flow) of the fluid streaming along the sphere. The constants of proportionality in these relations are of $O(1)$ (Ref. [3, Eq. (4)] noted $C_1 = 0.991$ for the first relation, and Ref. [4, Eq. (44)]

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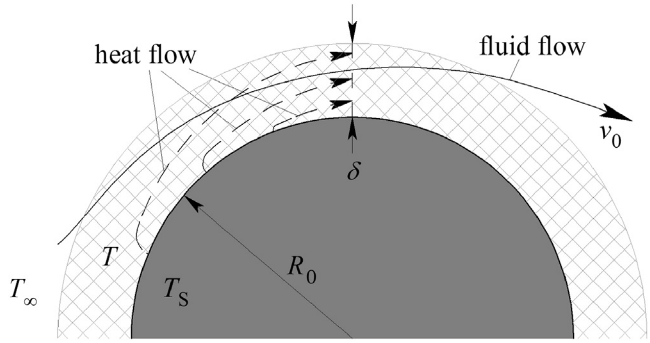

Fig. 1. Schematic cut showing the fluid flow circulating around a hot sphere (grey) of temperature T_S and radius R_0 and the resulting heat flow inside the (cross hatched) thermal boundary layer of thickness δ.

calculated $C_0 = 1.13$ for the second one). These results immediately raise several questions: Wherein exists the deeper connection between the energy loss P of an object immersed in a fluid stream, defined in Eq. (3) or (8), and the Péclet number (2) ? What determines the special rational number in the exponent of the Péclet number ? Depends it also on the shape of the object and the temperature field on its surface ? Which role plays the fluid flow field ?

In the following these questions are answered theoretically for the general 3-dimensional case of an incompressible fluid flow field of high Péclet number, streaming along an object of arbitrary shape and surface temperature field. Details are given in the next section. Even under these very general conditions the simple relation $Nu = C_k Pe^{1/(k+2)}$ with non-negative integer values k holds independently of the particular temperature field in the thermal boundary layer around the object. The value of k depends only on the boundary condition and the behavior of the fluid flow field near the object. For a shear flow with no-slip boundary conditions for example, which is mostly observed in practical applications, and for which the value $k = 1$ results (see Sec. 3), we find $Nu = C_1 Pe^{1/3}$. The validity of this simple relation has from a practical point of view the advantage, that C_k, which is a constant that contains all the complicated geometric details of the 3-dimensional temperature distribution and flow field near the object, can easily and accurately be determined experimentally and needs not to be calculated theoretically.

2. Formulation of the problem

The total steady heat transport in a moving medium relative to a fixed observer consists in a superposition of convective heat flow, i.e., the movement of heat containing fluid elements, and conductive heat flow, i.e., the diffusion of heat within in the fluid driven by a temperature gradient. The first mechanism is physically described by the convective heat flow density

$$\mathbf{j}_{mov}(\mathbf{x}) := \mathbf{v}(\mathbf{x})\rho c_p T(\mathbf{x}), \tag{4}$$

where the product of the mass density ρ, which is considered to be constant here, with the specific heat c_p (related to the mass) and the steady temperature field $T(\mathbf{x})$ in the fluid just corresponds to its local heat (energy) density, and where $\mathbf{v}(\mathbf{x})$ denotes its steady flow field. The second mechanism is physically described by the conductive heat flow density

$$\mathbf{j}_{con}(\mathbf{x}) := -\lambda(\mathbf{x})\nabla T(\mathbf{x}), \tag{5}$$

where the thermal conductivity of the fluid $\lambda(\mathbf{x}) = \lambda(T(\mathbf{x}))$ with $\lambda_\infty := \lambda(T_\infty)$, cf. Eq. (10), is a function of its temperature field. Neglecting dissipative heat generation by the viscous shear flow, the total heat in the fluid is conserved in the steady state. This means that locally

$$\nabla \cdot [\mathbf{j}_{mov}(\mathbf{x}) + \mathbf{j}_{con}(\mathbf{x})] = 0. \tag{6}$$

Mass conservation in the incompressible fluid implies analogously.

$$\nabla \cdot \mathbf{v}(\mathbf{x}) = 0. \tag{7}$$

For a solid object immersed in the fluid the total heat flow (power) from its surface $\partial S(0)$ into the surrounding fluid hence results in

$$\begin{aligned} P &:= \oint_{\partial S(0)} [\mathbf{j}_{con}(\mathbf{x}_S) + \mathbf{j}_{mov}(\mathbf{x}_S)] \cdot \mathbf{n}(\mathbf{x}_S) dS(\mathbf{x}_S) \\ &= -\oint_{\partial S(0)} \lambda(\mathbf{x}_S)\mathbf{n}(\mathbf{x}_S) \cdot \nabla T(\mathbf{x}_S) dS(\mathbf{x}_S), \end{aligned} \tag{8}$$

where it was assumed, that the component of the fluid flow field in normal direction $\mathbf{n}(\mathbf{x}_S)$ to the surface disappears in each point $\mathbf{x} = \mathbf{x}_S$ on the object surface, i.e.,

$$\mathbf{n}(\mathbf{x}_S) \cdot \mathbf{v}(\mathbf{x}_S) = 0. \tag{9}$$

This means, that on the object surface heat transfer into the surrounding fluid is performed by conduction only. For the temperature field of the fluid it is supposed, that on the object surface it corresponds to the surface temperature field $T_S(\mathbf{x}_S)$, and that "far away" from the object it assumes the constant value T_∞

$$T(\mathbf{x}_S) = T_S(\mathbf{x}_S) \quad \text{and} \quad \lim_{|x| \to \infty} T(\mathbf{x}) = T_\infty. \tag{10}$$

The Eq. (4)–(10) form the basic set of equations for a description of the general steady heat transfer problem in a given incompressible fluid flow field $\mathbf{v}(\mathbf{x})$.

In the present work the following conditions are assumed:

1) An incompressible fluid flow field $\mathbf{v}(\mathbf{x})$ circulates around a 3-dimensional object of surface temperature field $T_S(\mathbf{x}_S)$, as sketched in Fig. 2. The surface $\partial S(0)$ of the object is assumed to be convex and smooth enough (no edges), such that a curvature can everywhere be defined with the two orthogonal radii $R_{C1}(\mathbf{x}_S)$ and $R_{C2}(\mathbf{x}_S)$ being in the order of magnitude of the characteristic object dimension R_0

$$R_0 := \langle |\mathbf{x}_S| \rangle = \oint_{\partial S(0)} |\mathbf{x}_S| dS(\mathbf{x}_S) \Big/ \oint_{\partial S(0)} dS(\mathbf{x}_S). \tag{11}$$

2) Only the case of high Péclet numbers, defined in Eq. (2), is considered here, where, due to the low characteristic thermal conductivity λ_∞ of the fluid and the high characteristic speed v_0 of its flow along the object, the thermal energy, diffusing according to Eq. (8) from its surface $\partial S(0)$ in normal direction into the fluid, is tangentially "blown away", so that according to our considerations in the introduction the total heat transfer from the object into the fluid occurs in a thin thermal boundary layer of thickness $\delta(\mathbf{x}_S) \ll R_0$ only. Consequently the temperature