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On the bimodal nature of a confined buoyant plume. Part II: Flow structure echoes in state space

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ABSTRACT

This work represents the second part of a twofold study aiming at a detailed characterisation of the low order dynamics of a buoyant plume arising from an enclosed horizontal cylindrical heat source, for which chaos unfolds through a period doubling cascade preluded by a window of quasiperiodicity. In part I, this behaviour has been found to be related to the inherent bimodal nature of the flow, and an analytical characterisation of the asymptotic flows in the vicinity of the bifurcation points has been carried out.

In this second part, the thermo-fluid dynamic origin of this bimodal flow is investigated. A novel approach is proposed based on the observation of the existing correspondences between the recurrent formation of cellular flow structures, and the global structure of the system dynamics, as described by the attractor in an appropriate state space. In particular, the two modes are shown to correspond to the existence of two alternate and distinct sequences of flow patterns, and the locking between them corresponds to the synchronisation of analogous events in the evolution of the flow structures. Moreover, reported results show that, within the same framework, the period doubling cascade generated by the extinction of the quasiperiodic flow can be seen as a consequence of the modulation of the cellular flow structures.

In light of the remarkable similarities with experimental evidences of other buoyancy-driven flows, the proposed approach may represent a useful paradigm for the analysis of transitional, confined convection problems.

1. Introduction

Buoyancy-induced flow from an enclosed thermal source is a topic of great relevance for practically all applications involving passive heat transfer as the primary mean of thermal dissipation [1]. At the same time, it is also an interesting problem from the standpoint of thermofluids, since the phenomenology associated with this class of flows can encompass all the fundamental features of natural convection [2].

Such a variety can be easily figured out by considering the paradigmatic case of a cavity formed by an infinite square parallelepiped and a cylindrical heating source placed at its centre [3] (see Fig. 1): as soon as a temperature difference is applied between the cylinder and the enclosure walls, fluid motion ensues immediately around the cylinder. On the other hand, the fluid in the top part of the enclosure is subject to an unstable vertical gradient, as in the Rayleigh-Bénard problem, while vertical boundary layers are invariably forming at the enclosure sidewalls [4]. The combination of these situations in a single system can produce a number of different flow configurations and transition phenomena, depending on the set of values of the characteristic geometrical and thermo-fluid dynamic parameters [5–13]. In particular, if the gap H between the cylinder top and the enclosure ceiling is large, a thermal plume forms above the cylinder [14].

The connection between bifurcative flow patterns and transitional dynamical behaviours of confined thermal plumes was first investigated by Desrayaud and Lauriat [15], who performed numerical simulations for the case of a line heat source lying on the centerplane of an air-filled rectangular vessel. Depending on the depth of immersion of the source, they identified steady-state and unsteady symmetry-breaking patterns, associated with sub- and supercritical pitchfork and Hopf bifurcations. A similar study was carried out more recently by Bouafia and Daube [16], who retraced

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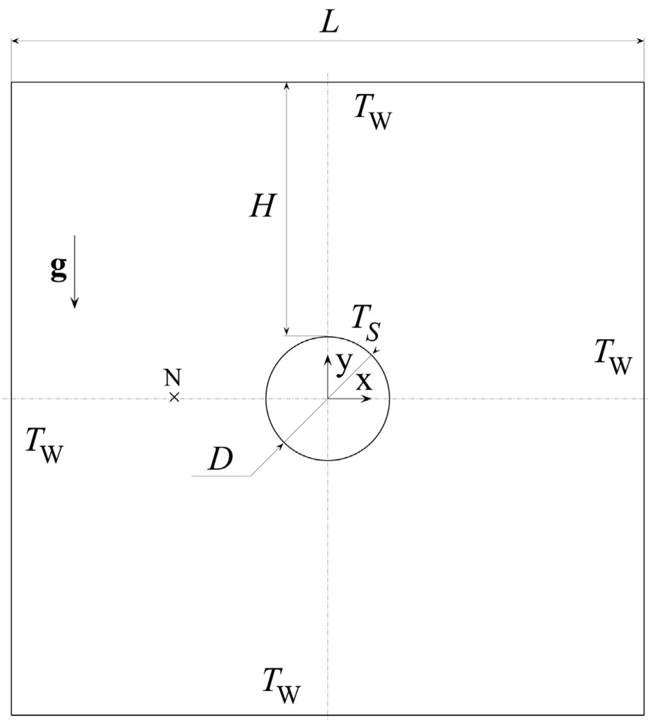

Fig. 1. Schematic view of the system under consideration.

the analyses of Desrayaud and Lauriat by substituting the line heat source with a square-sectioned cylinder. Results were in accordance with previous findings and substantiated them with a deeper insight on the transitional flows and thermal structures encountered. The entire route to chaos for a moderate-gap, differentially heated horizontal annulus filled with air at atmospheric pressure was tracked experimentally by Labonia and Guj [17], who reported an interesting succession of dynamical behaviours and flow structures, comprising forward and backward transitions from periodic to quasiperiodic regimes characterised by different oscillation modes. Very recently, in the work of Liao and Lin [13], the influence of the Prandtl number on flow stability was investigated for the same case considered here, albeit for a different value of the characteristic aspect ratio of the problem, $A = L/H$. There, the onset of different unstable regimes for a specific range of Rayleigh number values is documented, and the corresponding flow patterns are also reported, showing the variety of behaviours that can be observed in this kind of systems.

In part I of this work [18], following previous studies [14,19,20], the authors have discussed the peculiar dynamical features of the system of Fig. 1, throughout the transition from steady-state to chaotic flow, for a fixed value of the aspect ratio A, namely $A = 2.5$. In particular, the occurrence of quasiperiodicity preluding to a period-doubling route to chaos was analysed in deep detail adopting a non linear approach to the analysis of the system dynamics; this led to the observation that the system dynamics are determined by the existence and the nonlinear interaction of two autonomous fluid dynamic modes, which, depending on the value of the (leading parameter, i.e. the) Rayleigh number $Ra = g\beta(T_S - T_W) H^3 Pr/\nu^2$, may either act as independent, as it happens in the quasiperiodic dynamics, or lock together, giving rise to a subharmonic cascade. In this second part of the study, starting from the first unsteady, periodic regime, the whole route to chaos is revisited with a specific focus on the relationships between the system dynamics and the flow and thermal patterns. In particular, the emergence of the two separate modes, surviving throughout and beyond the whole transition scenario, is documented by a detailed analysis of the observed recurrence of ordered sequences of two-dimensional flow structures, together with the corresponding “echoes” of the evolution of such structures in the state space representation of the system dynamics. A proper state space has been defined by using the pointwise sampling of the field variables T, u_x and u_y at the location N reported in Fig. 1.

For brevity, the mathematical formulation of the problem and the description of the numerical method, already included in part I, are omitted here.

2. Results and discussion

The sequence of bifurcations outlined in Ref. [14] for increasing Ra is abridged here by the diagram of Fig. 2, reconstructed from the comprehensive database of 2D numerical simulations carried out in the frame of the present research. In particular, discrete points in the diagram correspond to intersections of the system trajectory in the state space T-u_x-u_y with a suitable Poincaré surface of section.

For the system under consideration, transition to chaos begins with a first supercritical Hopf bifurcation (not shown in Fig. 2, but fully discussed in Ref. [4]) at $Ra = 6.62 \times 10^4$, generating a periodic limit cycle P_1 out of the base flow, consisting of a stationary couple of counter-rotating cells sustaining a stable thermal plume which rises from the hot cylinder. Such a regime is characterised by a single dominant frequency f_1, and, hence, by a single periodicity. The P_1 cycle persists up until $Ra_{P1\to T2} \simeq 1.738176 \times 10^5$, where, after a Neimark-Sacker bifurcation, it loses its stability, giving rise to a quasiperiodic flow on a T_2 torus. The birth of the T_2 torus is associated with the appearance of a second independent frequency f_2, which, in the present case, is observed to lie very close to $f_1/2$ (see Ref. [14] and part I of the present work). At $Ra_{T2\to P2} \simeq 1.792782 \times 10^5$ the growth of the nonlinear coupling between the two frequencies f_1 and f_2 determines their locking, and a period-2 limit cycle (P_2) emerges. If Ra is increased further, the system undergoes a period-doubling cascade, that was tracked up to the seventh doubling, i.e. to the P_{128} cycle. Finally, for $Ra \geq 1.95025 \times 10^5$ the system exhibits chaotic behaviour.

The aim of the present study is to associate the different dynamical behaviours outlined above with the corresponding evolution of the flow structures within the cavity. These consist of convective rolls that are created and destroyed due to and in mutual interaction with the swaying motion of the thermal plume with respect to its upright position along the vertical enclosure axis.

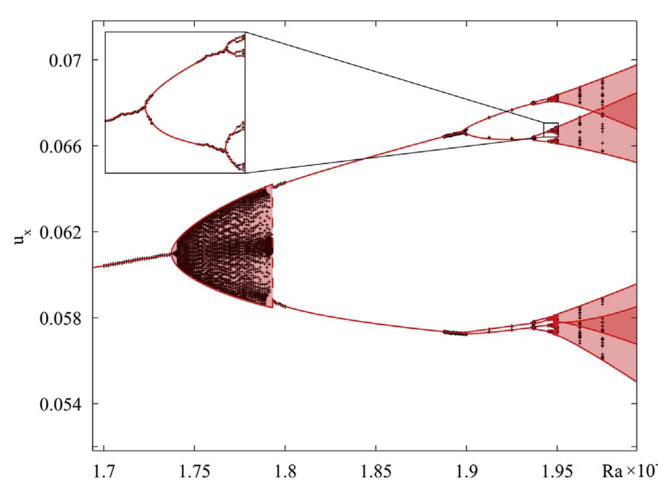

Fig. 2. Bifurcation diagram reconstructed from Poincaré sections belonging to the database presented in Ref. [14], spanning the whole transition from periodic to chaotic flow.