Applied Energy 134 (2014) 374–381

Contents lists available at ScienceDirect

Applied Energy

journal homepage: www.elsevier.com/locate/apenergy

Analytic thermoelectric couple optimization introducing Device Design Factor and Fin Factor

J. Mackey [a,*], A. Sehirlioglu [b], F. Dynys [c]

[a] Department of Mechanical Engineering, University of Akron, Akron, OH 44325, USA
[b] Department of Materials Science and Engineering, Case Western Reserve University, Cleveland, OH 44106, USA
[c] NASA Glenn Research Center, Cleveland, OH 44135, USA

HIGHLIGHTS

- We introduce a Device Design Factor to account for thermal losses of a thermoelectric couple.
- We introduce a Fin Factor to account for lateral heat transfer in a thermoelectric couple.
- Conversion efficiency is analytically derived in terms of new design parameters.

ARTICLE INFO

Article history:
Received 24 April 2014
Received in revised form 5 August 2014
Accepted 7 August 2014

Keywords:
Thermoelectric modeling
Seebeck coefficient
Peltier effect

ABSTRACT

An analytic solution of a thermocouple has been developed in order to gain a deeper understanding of the physics of a real device. The model is established for both rectangular and cylindrical couples and is made to account for thermal resistance of the hot and cold shoes and lateral heat transfer. A set of dimensionless parameters have been developed to determine couple behavior and serve as simplifying justifications. New dimensionless parameters, Device Design Factor and Fin Factor, are introduced to account for the thermal resistance and lateral heat transfer, respectively. Design guidelines on couple length and cross-sectional area have been established to account for conditions encountered by a realistic couple. As a result of thermal resistances a lower limit on the length of the couple can be established. In the case of a lateral heat transfer couple the efficiency is found to depend upon cross-sectional area of the leg in such a fashion as to suggest the need to design large area couples. The classic thermoelectric solution neglects the effect of thermal resistance and lateral heat transfer, leading to an over estimated conversion efficiency. The work presented provides a path to incorporate these neglected factors and offers a simplified estimation for couple performance based on analytic solutions of governing equations.

1. Methodology

The foundation of the thermoelectric device is the fundamental thermoelectric couple, composed of two conducting legs classically configured electrically in series and thermally in parallel. The coupled transport of thermal and electrical energy through the legs serves to operate as a heat pump or heat engine depending on the driving mechanism. Therefore it is of critical interest to have a clear understanding of the physics of a thermoelectric couple in order to design both materials and devices capable of providing the desired thermoelectric output in an efficient and optimized manner.

The governing equations of a couple can be derived from applying the generalized thermoelectric form of Ohm's and Fourier's Laws to a control volume, obtainable from the conjugate flux force relations of Onsager [1,2]. A similar approach can be found in the classic conduction equation in many heat transfer texts, such as Arpaci [3], Ozisik [4], and Carslaw and Jaeger [5]. The couples were investigated as steady state one dimensional domains, neglecting temperature and electrical gradients in directions other than the primary leg length, and with constant material properties. Steady state is justified by common application of devices, and for the purpose of obtaining concise solutions. Investigation into transient behavior has been performed by Meng et al. [6], Alata et al. [7], and Montecucco et al. [8] among others. The one dimensional assumption can be justified by investigating Biot numbers for the primary leg direction versus the lateral direction of typical thermoelectric legs. The constant material properties simplification was

* Corresponding author. Tel.: +1 (216)433 3901.
E-mail address: jam151@zips.uakron.edu (J. Mackey).

http://dx.doi.org/10.1016/j.apenergy.2014.08.034

used to form a linear model so that meaningful solutions could be developed. The effect of variable material properties is out of the scope of this work. Some of the effects of variable material properties may be accounted for by incorporating the standard integral-averaged approach for the calculation of the typical thermoelectric properties, see Sandoz-Rosado and Stevens [9].

The resulting system model consists of four coupled ordinary differential equations and a fifth coupled algebraic Eqs. (1)–(3). Eq. (1) gives the thermal governing equations for two legs, **a** and **b**, while Eq. (2) gives the electrical governing equations for the same legs. The fifth equation (Eq. (3)) represents Ohm's Law.

$$-\frac{d^2\widehat{T}_{a,b}}{d\hat{x}^2}+\beta_{a,b}\widehat{I}_{a,b}\frac{d\widehat{T}_{a,b}}{d\hat{x}}-\gamma_{a,b}\widehat{I}^2_{a,b}=0, \tag{1}$$

$$\frac{d\hat{\phi}_{a,b}}{d\hat{x}}=-\xi_{a,b}\frac{d\widehat{T}_{a,b}}{d\hat{x}}-\lambda_{a,b}\widehat{I}_{a,b}, \tag{2}$$

$$\hat{\phi}_b(\hat{x}=1)-\hat{\phi}_a(\hat{x}=1)=\widehat{I}_b. \tag{3}$$

The complete model for a rectangular couple with insulated leg sides and two legs electrically in series, **a** and **b**, is described in terms of dimensionless parameters. The assumed dimensionless variables of the model include a space coordinate $\hat{x}=x/L$ normalized with leg length, temperature $\widehat{T}=T/\Delta T$ normalized with the temperature difference driving the Seebeck effect, voltage $\hat{\phi}=\phi/(\Delta\alpha\cdot\Delta T)$ normalized with a characteristic couple open circuit voltage due to couple Seebeck coefficient $\Delta\alpha=\alpha_b-\alpha_a$ and electrical current $\widehat{I}=IR/(\Delta\alpha\cdot\Delta T)$ normalized with characteristic load resistance R and open circuit voltage. Substitution of the dimensionless parameters into the governing equation results in a set of characteristic parameters which govern the behavior of all couples and provide validation for assumptions. The effect of Thomson heat is captured in the dimensionless parameter $\beta=\tau\Delta\alpha\Delta TL/(ARk)$, which serves as a measure of the accuracy of neglecting Thomson heat τ, in terms of couple Seebeck $\Delta\alpha$, operating temperature difference ΔT, length L, cross sectional area A, load resistance R, and thermal conductivity k. This heat is classically neglected as part of the assumption of constant material properties by the second Kelvin relation $\tau=T\frac{d\alpha}{dT}$ [1,2]. Work in accounting for this heat has been performed by Sherman et al. [10], Yamashita [11], and Min et al. [12]. For β values much smaller than unity the assumption of neglecting Thompson heat is valid, but in the case of a couple with β on the order of unity this assumption is no longer justified. Notice the term involves not only material properties but also a geometric slenderness ratio L/A as well as operational parameters ΔT and R. The effect of Joule heating is captured by the dimensionless parameter $\gamma=\Delta\alpha^2\Delta TL^2/(A^2R^2k\sigma)$, with the introduction of electrical conductivity σ. The dimensionless voltage due to Seebeck effect is captured by $\xi=\alpha/\Delta\alpha$, and the voltage due to electrical losses is found in $\lambda=L/(AR\sigma)$.

Included in the model are the eight boundary conditions required for complete specification of the problem statement. Under a strict set of simplifying assumptions the model can be seen to return the same solution classically offered as the analytic solution of a thermoelectric couple, see Sherman et al. [10], Lampinen [13], or Bejan [14]. These assumptions include: constant isotropic material properties, fixed hot and cold shoe temperatures, insulated leg sides, steady state, and one dimensional conduction. Additionally, due to the electrical series nature of the couple the electrical current in legs **a** and **b** are equal in magnitude and opposite in direction. With appropriate application of aforementioned boundary conditions and solution of the coupled equations the classic solution of the couple can be found.

Calculation of the couple's thermodynamic conversion efficiency reveals two critical design parameters, well suited for optimization of device design: (i) the geometric factor $X=A_bL_a/(A_aL_b)$ is the ratio of the leg slenderness for **a** and **b** legs and (ii) the load factor $Y=R/(L_a/(A_a\sigma_a)+L_b/(A_b\sigma_b))$ is a ratio of load resistance to couple resistance. Details of the classic optimization can be found in literature and provide a means of calculating the optimized X and Y values to obtain a couple with maximum conversion efficiency [10,13–15]. In addition to the device design optimization, the traditional thermoelectric figure of merit can be extracted from this conversion efficiency and used as a material design guideline.

Similarly the method can be applied to a cylindrical couple configured with radial heat transfer. *Legs in a cylindrical couple are configured as a solid washer or ring shape with the temperature gradient ranging from the inside to outside radius*. Sets of p- and n-type washers can be electrically connected in series to complete a circuit (Fig. 1), similar to their rectangular counterparts. Cylindrical couples may prove to be well suited for the design of compact heat exchangers which require a radial conduction path. For instance, the cylindrical couple would serve well in an energy harvesting application of a coolant line passing through a hot exhaust chamber. The outside radius of the legs would be in communication with the hot ambient thermal reservoir, and the cold junction would be in direct thermal contact with a coolant fluid. Cylindrical couples have been theoretically and experimentally investigated by Min and Rowe [16,17], Landecker [18], Lund [19], and Liu [20]. The previous governing equations Eqs. (1)–(3) can be substituted for their cylindrical equivalents and solved, see Appendix A. The temperature and voltage profiles, now functions of radius r and leg width w, can be used to calculate the couple's thermodynamic conversion efficiency. Again, two design parameters, with similar physical meaning to their rectangular counterparts, can be extracted. The appropriate geometric factor as derived by this work becomes $X=w_b\ln\frac{r_{o,a}}{r_i}\Big/\left(w_a\ln\frac{r_{o,b}}{r_i}\right)$ with subscript **o** indicating outside radius and **i** inside radius. Likewise the new cylindrical load factor derived in this work becomes $Y=R\Big/\left(\ln\frac{r_{o,a}}{r_i}/(2\pi\sigma_aw_a)+\ln\frac{r_{o,b}}{r_i}/(2\pi\sigma_bw_b)\right)$. Optimization of the efficiency in terms of the cylindrical X and Y design factors is identical to the rectangular case and results in the same final optimized values, as suggested by Min and Rowe [16]. The similarity of the rectangular and cylindrical solutions is expected and reported in literature,

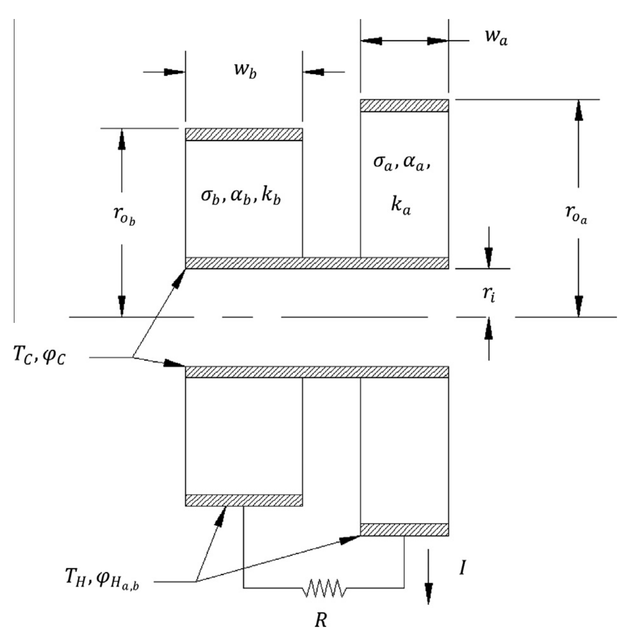

Fig. 1. Schematic of a cylindrical couple, highlighting the geometric parameters of interest leg width w, and inner and outer radius r_i and r_o.