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Natural convection of power law fluids in inclined cavities

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ABSTRACT

Steady two-dimensional natural convection in rectangular two-dimensional cavities filled with non-Newtonian power law-Boussinesq fluids is numerically investigated. The conservation equations of mass, momentum and energy are solved using the finite volume method for varying inclination angles between 0° and 90° and two cavity height based Rayleigh numbers, $Ra = 10^4$ and 10^5, a Prandtl number of $Pr = 10^2$ and three cavity aspect ratios of 1, 4 and 8. For the vertical inclination of 90°, computations were performed for two Rayleigh numbers $Ra = 10^4$ and 10^5 and three Prandtl numbers of $Pr = 10^2$, 10^3 and 10^4. In all of the numerical experiments, the channel is heated from below and cooled from the top with insulated side walls and the inclination angle is varied. A comprehensive comparison between the Newtonian and the non-Newtonian cases is presented based on the dependence of the average Nusselt number $\overline{Nu}$ on the angle of inclination together with the Rayleigh number, Prandtl number, power law index n and aspect ratio dependent flow configurations which undergo several exchange of stability as the angle of inclination ϕ is gradually increased from the horizontal resulting in a rather sudden drop in the heat transfer rate triggered by the last loss of stability and transition to a single cell configuration. A correlation relating $\overline{Nu}$ to the power law index n for vertically heated cavities for the range $10^4 \le Ra \le 10^5$ and $10^2 \le Pr \le 10^4$ and valid for aspect ratios $4 \le \text{AR} \le 8$ is given.

1. Introduction

Buoyancy driven Newtonian and non-Newtonian flows in rectangular enclosures are found in a variety of engineering applications such as oil-drilling, pulp paper, slurry transport, food processing and polymer engineering. Pseudoplastic fluids are used in compact heat exchangers or electronic modules as a cooling enhancing medium. For differentially heated two-dimensional enclosures with adiabatic side walls, the heat transfer characteristics are influenced by the inclination of the cavity with respect to the horizontal plane, Prandtl number, and the Rayleigh number based on the height of the cavity. Although the case of Newtonian liquid has received considerable attention (see Gebhart et al. [1], Ostrach [2] and Khalifa [3] for reviews), there is only a limited number of articles dealing with the non-Newtonian case.

For the Newtonian case, flows in such configurations have been the subject of several experimental and numerical studies. Catton et al. [4] and Arnold et al. [5] investigated experimentally and numerically heat transfer in inclined cavities for a range of aspect ratios, Rayleigh numbers and angles of inclination. Ozoe and Sayama [6] and Ozoe et al [7,8] experimentally investigated and numerically computed values of the Nusselt number for natural convection heat transfer in square and rectangular channels. They note the existence of several modes of two-dimensional roll cells in the flow field as the angle of inclination is gradually increased from the horizontal position. The angle of transition between modes depended upon the value of the Rayleigh number and the aspect ratio with the Nusselt number showing a discontinuous behavior. The transition of flow modes was also studied by Soong et al. [9] who noted the influence of initial conditions on the flow pattern formation. Corcione [10], considered the effect of bi-directional differential heating in horizontal cavities of several aspect ratios for Rayleigh numbers between 10^3 and 10^6 and conjectured that the increase in the number of roll cells occurring as the aspect ratio increased may be explained through the progressive breakdown of the density stratification in the fluid layers adjacent to the top and bottom walls that bring the formation of hot and cold fluid streams moving upward and downward across the cavity with direct effect on the temperature distribution. Ozoe et al. [8] and Soong et al. [9] and Wang and Hamed [11] have all demonstrated flow mode transition and hysteresis phenomenon for Rayleigh numbers greater than 3000. For a range of Rayleigh numbers up to 10^4, the latter conducted a systematic numerical study of the variation of the Nusselt number with angle of inclination and concluded that

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this type of flow family could have dual or multiple solutions due to the effect of initial conditions. They confirm the dependence of the solution on the initial conditions and clarify further the successive loss of stability with increasing angle of inclination from the horizontal, each successive loss of stability leading to flow configurations with smaller number of vortices with the final bifurcation leading to a single cell configuration. The last transition from a multiple cell to a single cell configuration is also associated with a discontinuity in the Nusselt number. The size of the jump in the value of the Nusselt number is dependent on the aspect ratio and the Rayleigh number. The transition to a single cell configuration occurs at gradually larger angle of inclinations with growing aspect ratios. The size of the discontinuity with growing Rayleigh numbers seems to be getting smaller for aspect ratios smaller than 12, however for aspect ratio 12 the size of the discontinuity seems to increase with increasing Rayleigh numbers.

When it comes to non-Newtonian liquids there are relatively few references in the literature. It appears that the numerical study by Ozoe and Churchill [12] aimed at determining the threshold for the onset of Rayleigh–Benard convection in power law fluids was one of the first in the field. The critical Rayleigh number was found to increase with the power index. However the results showed a tendency to give exaggerated values when compared to the experimental and theoretical data reported by Tien et al. [13]. More recently, Kim et al. [14] considered transient buoyant convection in a square cavity subjected to hot and cold temperature on the vertical side walls for Newtonian and non-Newtonian shear-thinning power law fluids of the Ostwald-De Waele type. The study concludes that for high Rayleigh $Ra = 10^5$–10^7 and Prandtl numbers $Pr = 10^2$–10^4, convective activity intensifies with decreasing power law index n resulting in enhanced overall heat transfer coefficients. Ohta et al. [15] studied numerically transient heat transfer in a square cavity heated from the bottom and cooled from the top using the Sutherby model for shear-thinning fluids, such as aqueous solutions of Natrosol 250H hydroxyethyl cellulose and found that shear-thinning resulted in larger heat transfer rates than Newtonian fluids. Their study reveals as well that for highly pseudoplastic fluids and for a large Rayleigh number equal to 10^5 complex flow patterns consisting of unstable multiple roll cells are generated leading to an oscillating Nusselt number with time.

Thermal convection of micro-emulsion slurry, which exhibits non-Newtonian power law characteristics, was studied numerically and experimentally by Inaba et al. [16] in rectangular cavities. They found that heat transfer rate is increased with the introduction of shear-thinning. Flow and heat transfer in a shallow rectangular enclosure filled with Ostwald-De Waele liquids heated from the side under a constant heat flux assumption is studied analytically and numerically by Lamsaadi et al. [17]. They determined that if the aspect ratio and Prandtl numbers are large enough (>100) the flow and heat transfer rate characteristics become independent of any increase in these parameters and the flow is essentially controlled by the Rayleigh number and the power law index.

In summary, it seems that apart from the study of Kim et al. [14] which was limited to a square horizontal cavity with shear-thinning fluids, no study is available on thermal natural convection of non-Newtonian power law fluids in rectangular two-dimensional tilted enclosures heated from below and cooled from above under a constant wall temperature assumption. The main goal of this article is hence to fill this gap and study the effect of shear-thinning and shear-thickening on heat transfer rate using the power law model of Ostwald-De Waele fluids and compare the trends in the flow structure changes to those of a Newtonian fluid with a high Prandtl number in the same configuration with varying angle of inclination and, to the extent possible, provide correlations relating $\overline{Nu}$ to Ra, Pr and n for a range of aspect ratios.

2. Mathematical formulation

A two-dimensional rectangular cavity filled with a non-Newtonian fluid is considered. The inclination angle of the cavity φ varies between $0^\circ \le \varphi \le 90^\circ$. The aspect ratio is AR = L/H, the ratio of the length L of the isothermal walls to the length H of the adiabatic walls. The top (cold) and the bottom (hot) surfaces of the cavity are maintained at constant temperatures T_c and T_h, while the two side walls are kept adiabatic as shown in Fig. 1. Flow in the cavity is assumed laminar, steady and two-dimensional. The Boussinesq assumption is used and viscous dissipation is assumed to be negligible. The buoyancy force is caused only by the density gradient, thus:

$$\frac{\rho}{\rho_0} = 1 - \beta(T - T_0) \tag{1}$$

where β is the coefficient of thermal expansion, ρ is the fluid density at temperature T and ρ_0, T_0 are the corresponding reference values, respectively. The field conservation equations of mass, momentum and energy are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \frac{1}{\rho_0}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right) + g\beta(T - T_0)\sin\phi \\ u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} &= -\frac{1}{\rho_0}\frac{\partial p}{\partial y} + \frac{1}{\rho_0}\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}\right) + g\beta(T - T_0)\cos\phi \end{aligned} \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

Where the velocity vector is expressed in terms of its Cartesian components (u,v) along the x and y directions of the coordinate system shown in Fig. 1; p, ν,κ and g represent the pressure, the kinematic viscosity, the thermal diffusivity and the acceleration of gravity, respectively.

The fluid obeys the non-Newtonian power law given by:

$$\tau_{ij} = \mu_a D_{ij} = Ke^{T_0/T}\left(\frac{1}{2}D_{kl}D_{kl}\right)^{\frac{(n-1)}{2}} D_{ij} \tag{5}$$

where the rate of strain is given by $D_{ij} = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)$ and K and n are the consistency and the power law indices, $n < 1$ defines shear-thinning, $n = 1$ and $n > 1$ correspond to the Newtonian and to shear-thickening fluids, respectively.

Since viscosity is no longer a constant fluid property but depends on shear rate, appropriate Prandtl and Rayleigh numbers for power law fluids need to be defined. Several approaches have been adopted in the past [18–22]. In [14,19], the introduction of a physical quantity with dimensions of $(\text{length})^2(\text{time})^{-1}$ to play

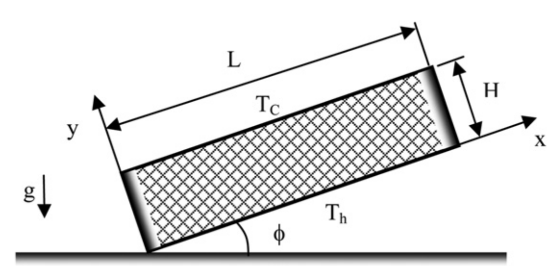

Fig. 1. Geometrical configuration.