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Effects of the thick walled pipes with convective boundaries on laminar flow heat transfer

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HIGHLIGHTS

- Analytical approach is employed to solve the energy equations with convective boundary condition of the third kind.
- The wall thickness, Bi and k_{pf} significantly influence the interfacial heat flux, the wall and fluid bulk temperatures.
- The fluid bulk and wall temperatures decrease with decreasing pipe wall thickness and increasing Bi number and k_{pf}.
- Increase in the convective heat loss corresponds to a decrease in wall thickness but increase in both Bi and k_{pf}.
- The thermal entrance length increases with pipe wall thickness while it decreases with increase in both Bi and k_{pf}.

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ABSTRACT

Conjugate heat transfer in laminar tube flow with convective boundary conditions is considered analytically. The steady state problem involving two-dimensional wall and axial fluid conduction is solved using separation of variables for a thick walled cylindrical pipe. The effects of the wall thickness, external Biot number and wall-to-fluid thermal conductivity ratio are investigated on the heat flux, fluid bulk and wall temperatures. Results are presented for the cases when the wall thickness is between 0.1 and 2, Biot number ranging between 0.1 and 10, and the ratio of wall-to-fluid thermal conductivity between 3 and 100. These parameters are found to significantly affect the heat transfer characteristics at the thermal entrance region, for instance, increase in wall thickness results in reduced heat flux while increase in Biot number and the ratio of the wall-to-fluid thermal conductivity result in increased heat flux. Decrease in wall thickness, increase in both Biot number and the ratio of the wall-to-fluid thermal conductivity correspond to decreased fluid bulk and wall temperature profiles.

1. Introduction

Conjugate heat transfer in circular and rectangular ducts received considerable attention in the last century since the formulation of the Graetz problem. For the Graetz problem and subsequent studies, it was common practice to impose heat flux or temperature boundary conditions at the fluid-wall interface as well as neglect the duct wall conduction in the heat transfer process. In most realistic situations, the boundary conditions at the interface are not known initially but depend on the coupling between convection and conduction mechanisms at the interface [1]. It is the coupling between convection in the fluid and conduction in the duct that give the problem the name conjugate. This heat transfer problem is better analyzed by considering the simultaneous heat transfer inside the fluid and the wall. A comprehensive review of studies conducted on heat transfer in conventional ducts was carried out by Shah and London [2] and, Shah and Bhatti [3]. They concluded that wall conduction might have a significant effect on heat transfer especially in the thermal entrance region.

In earlier studies on conjugate heat transfer, various analytical and numerical solutions were employed to solve both the problems of thermal entrance region with axial conduction term and of fully developed flow. Most of these studies have also considered either prescribed heat flux or wall temperature boundary conditions. However, limited studies have been done on the problems of convective boundaries, that is, problems involving boundary conditions of the third kind [4–6].

Among the earlier studies on conjugate heat transfer are Mori et al. [7,8], who considered the effect of wall conduction between parallel plates and in circular pipes for uniform heat flux and

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Nomenclature

Symbols

A	area (m^2)
Bi	Biot number
C_p	specific heat at constant pressure (kJ/kg °C)
d	pipe diameter (m)
G	dimensionless function of axial position in fluid eq.
h	heat transfer coefficient (W/m^2 °C)
J_0	Bessel function of first kind of zero order
J_1	Bessel function of first kind of unity order
k	thermal conductivity (W/m °C)
L	characteristic length of system/pipe (m)
Nu	Nusselt number, $h_i d_i/k_f$
Nu_m	modified nusselt number, $U_i d_i/k_f$
Pe	Peclet number
q'	dimensionless heat flux
r	radius (m)
R_{Total}	total thermal resistant (°C/W)
R_0	eigenfunction of order zero
R_1	eigenfunction of order unity
T	temperature (°C)
T_∞	ambient (free stream condition) temperature (°C)
U	overall coefficient of heat transfer (W/m^2 °C)
$u(\rho)$	is a function of dimensionless radius, $1-\rho^2$
v	velocity (m/s)
x	position in the axial direction (m)

Greek symbols

ξ	dimensionless position in axial direction
η	dimensionless pipe thickness
ϕ	dimensionless function of radius in fluid eq.
ψ	dimensionless function of radius in pipe eq.
Ψ	ratio of outlet to inlet temperatures
χ	dimensionless function of axial position in pipe eq.
λ	characteristic eigenvalues or roots of fluid eq.
ω	characteristic eigenvalues or roots of pipe eq.
ρ	dimensionless radius
Θ	dimensionless temperature

Subscript

b	bulk
e	exit
f	fluid
i	inner number of finite roots in pipe eq.
j	number of finite roots in fluid eq.
m	mean
o	outer
p	pipe
pf	pipe wall-to-fluid
w	wall
k	an arbitrary integer

constant surface temperature boundary conditions. The effect of axial wall conduction between parallel plates was analyzed for Couette flow by Davis and Gill [9]. Faghri and Sparrow [10] in their study on simultaneous wall and fluid axial conduction in laminar pipe flow proposed criteria for judging the importance of the axial heat conduction. Like Faghri and Sparrow, Zariffeh et al. [11] employed finite difference for their solution while Campo and Rangel [12] used analytical methods in their study of conjugate effect of one-dimensional fluid and wall axial conduction. In all these studies, extremely thin ducts were assumed.

With the emerging applications of heat transfer in micro- and mini-channels in micro-electro-mechanical systems (MEMS) – in which wall thickness of the duct or pipe is significant, it is however reasonable to view the problems of conjugate heat transfer as a two-dimensional wall (radial and axial) conduction. For this reason, a thick-walled duct is better used for this analysis. Before the emergence of microtubes and channels though, some earlier researchers on conventional heat transfer in large tubes have extended their analysis to involve two-dimensional wall conduction. This found ready applications in high temperature, high pressure conveyance of fluid such as crude oil in the deep and ultra-deep offshore environment. Pagiliarini [13] and Barozzi and Pagiliarini [14] investigated analytically, flows in thick-walled ducts/pipes with two-dimensional wall conduction. Campo and Shuler [15] employed lumped system to analyze the simultaneous wall and fluid axial conduction in laminar pipe flow heat transfer. Bilir [16] employed finite difference method to solve the combined effect of two-dimensional wall and fluid conductions for low Peclet number ($Pe \leqslant 20$) laminar flow. He considered a thick-walled two regional large cylindrical pipe with external constant temperature and a change at a given section. Chung and Sung [17] employed direct numerical simulation for turbulent flow in concentric annulus for $Re = 8900$, $Pr = 0.71$, radius ratio of 0.1 and 0.5 and heat flux ratio of 1:10. Results revealed that vortex regeneration between the inner and outer walls caused higher thermal structure at the outer walls. In the study of the effect of numerical simulations on the heat transfer of a fully developed turbulent pipe flow with isoflux on the wall for $Re = 5500$, Redjem-Saad et al. [18] observed that for $Pr \geqslant 0.2$, temperature and turbulent heat flux increased with increasing Pr. Esfahani and Shahabi [19] investigated the effect of heat flux distribution on entropy generation. The results indicated that heat flux distribution affected the extent of entropy generation and that it could be regulated by varying heat flux distribution or its rate of change. Tso et al. [20] considered non-Newtonian fully developed laminar heat transfer in fluids between fixed parallel plates. The plates were maintained at different constant heat flux. The results showed that the power indices of the fluids and the viscous dissipation affected the heat transfer. Ates et al. [21] showed that wall thickness, wall-to-fluid thermal conductivity ratio, wall-to-fluid thermal diffusivity, Biot and Peclet numbers significantly influence the heat transfer characteristics in a thermally developing laminar flow in a two-dimensional wall and fluid conduction. The effect of the thickness of a trapezoidal wall placed between a heat source and a cold fluid was investigated on the hot spot temperature of the system [22]. ANSYS FLUENT 12.01 was used to optimize the thickness of the plate and it was shown that at the optimum thickness, the hot spot temperature decreased up to 25.06%. For the six cases of non-uniform heat flux supplied to a circular pipe flow with Prandtl number of 13,400, Al-Maliky [23] reported an increase in Nusselt number (for known Prandtl number) and an increase in maximum velocity at the center of the fluid as Reynolds number increased. Furthermore, correlations were developed for each case.

For application to micro-tube, Zhang et al. [24] considered conjugate effect of two-dimensional wall conduction and fluid axial conduction for simultaneous developing laminar flow and heat transfer in microtube with varying dimensionless wall thickness and constant outer surface temperature. Results revealed that the

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