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Mathematical modeling and numerical results of power-law fluid flow over a finite porous medium

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ABSTRACT

This paper presents a mathematical model and corresponding numerical results for a power-law fluid flowing in a channel partially filled with a homogeneous and isotropic porous medium. At the interface between the clear fluid and the porous material, a model for the stress jump condition takes into consideration the behavior of a power-law fluid. This study shows that the use of a modified permeability, K^*, satisfactorily describes the friction factor of the flow for $\mathrm{Re}_{\eta^*} \leq 1$ (Darcy regime). The mathematical modeling presented, supported by comparisons with analytical and numerical results, also shows that the form drag must be taken into account in the momentum equation, even for a power-law fluid. The mathematical modeling presented has been used to simulate Newtonian as well as power-law fluids flowing in both porous and unobstructed media. For a channel partially filled with porous material and under a fixed mass flow rate, results indicated that the pressure drop is a function of porosity, Darcy number, shear jump coefficient, β, and flow behavior index, n.

1. Introduction

In recent years, the research about flow of non-Newtonian fluids has received considerable attention due to its wide application in industry, for example, food engineering, petroleum production with enhanced oil recovery, chemical process such as distillation towers, and plastic processing [1]. In many cases, the flow occurs in ducts or channels fully or partially filled with blocks of porous material, *e.g.*, groundwater flow, filtration, ceramic processing, compact heat exchangers and many other engineering applications [2]. Thus, the correct description of the non-Newtonian fluid flow in the channel partially filled with porous material is of the paramount importance for improved understanding of various phenomena that occur in industrial processes.

The work of Shenoy (1994) [3] presented a broad literature review about Non-Newtonian fluid flow and heat transfer in a porous medium. He suggested, the use of the modified permeability in the Darcy term as function of the tortuosity factor, and that Forchheimer term can stay unchanged in the momentum equation. Hayes et al. (1996) [4], using volume averaging, obtained a similar momentum equation. Malin (1997) [5] proposed a modification to the damping function that showed to improve predictions for non-Newtonian fluids. Inoue and Nakayama (1998) [6] investigated, using modified permeability based on the solid particle diameter of the porous material, the viscous and inertia effects in pressure drop in non-Newtonian fluid flow across a porous medium. The authors obtained an expression for porous inertia that possesses the same function as Ergun's, but the value has indicated to be only one third of Ergun's. The works [7–10] studied the power-law fluid flow in a porous medium, using the modified permeability (suggested in Ref. [3]) in the momentum equation.

Ochoa-Tapia and Whitaker (1995a,b) [11,12] proposed an analytical expression to take into account the variation of the shear stress jump at the interface between clear fluid and porous medium. This approach produces a jump in the stress and presents a parameter that should be determined experimentally. Kuznetsov (1996–99) [13–16] used the boundary condition proposed by Ochoa-Tapia and Whitaker (1995a,b) [11,12] to obtain an analytical solution for a Newtonian fluid flow in a channel partially filled with porous material. The analytical development presented in Chandesris and Jamet (2006) [17] shows that the stress jump condition is also related to the pressure gradient and, they suggest a methodology to derive boundary conditions between a free fluid

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Nomenclature

A_i	Area of the interface fluid/solid [m^2]
Da	Darcy number, $Da = K/H^2$
f_{η^*}	Friction factor for power-law fluid, $f_{\eta^*} = 1/\mathrm{Re}_{\eta^*}$
$\mathbf{g}$	Gravity acceleration vector [m/s^2]
H	Distance between channel wall and symmetry line [m]
K	Permeability [m^2]
K^*	Modified permeability, $K^* = K^{\frac{(n+1)}{2}}/\phi^{(1-n)}$
L_c	Channel length [m]
m	Fluid consistency coefficient, $m = \rho \overline{u}_D^{(2-n)} H^n/\mathrm{Re}_H$ [Pa.s^n]
n	Flow behavior index
p	Thermodynamic pressure [Pa]
$\langle p \rangle^i$	Intrinsic (fluid) average of pressure p [Pa]
$\boldsymbol{R}$	Total drag force per volume unity volume [Pa/m]
Re_H	Reynolds number based on H, $\mathrm{Re}_H = \rho \overline{u}_D H/\mu$
Re_{η^*}	Modified Reynolds number based on η^*, $\mathrm{Re}_{\eta^*} = \rho \left| \overline{u}_D \right| \sqrt{K}/\eta^*$
S_φ	Source term
u_D	Component of Darcy velocity along $\overline{\eta}$ (normal) direction [m/s]
u_∞	Velocity at symmetry line [m/s]
u	Dimensionless velocity, $u = u_D/\overline{u}_D$
U	Dimensionless velocity, $U = u_D/u_\infty$
$\overline{u}_D$	Average Darcy velocity, $\overline{u}_D = 1/H \int_0^H u_D \mathrm{d}y$ [m/s]
$\mathbf{u}_i$	Microscopic velocity of the interface fluid/solid [m/s]
u_{D_P}	Darcy velocity component parallel to the interface [m/s]
$\mathrm{u_i}$	Dimensionless velocity at interface clear fluid/porous medium
$\mathbf{u}$	Microscopic velocity [m/s]
$\langle \mathbf{u} \rangle^i$	Intrinsic (fluid) average of $\mathbf{u}$ [m/s]
$\mathbf{u}_D$	Darcy velocity vector, $\mathbf{u}_D = \phi \langle \mathbf{u} \rangle^i$ [m/s]
x, y, z	Cartesian coordinates
Y	Dimensionless transverse coordinate, $Y = y/H$

Greek symbols

β	Shear stress jump coefficient
η	Apparent viscosity, $\eta = m\left|\sqrt{\left\{\left[\nabla \mathbf{u} + (\nabla \mathbf{u})^{\mathbf{T}}\right] : \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^{\mathbf{T}}\right]\right\}/\mathbf{2}}\right|^{n-1}$ [Pa.s]
η_D	Dimensionless apparent viscosity, $\eta_D = \left|(1/\phi)(\mathrm{d}u/\mathrm{d}Y)\right|^{n-1}$
η^*	Modified apparent viscosity, $\eta^* = m(\left|\overline{u}_D\right|/\phi\sqrt{K})^{n-1}$ [Pa.s]
η_{PM}	Apparent viscosity in the porous medium $\eta_{PM} = m\left|(\mathbf{1}/\phi)\sqrt{\left\{\left[\nabla \mathbf{u_D} + (\nabla \mathbf{u_D})^{\mathbf{T}}\right] : \left[\nabla \mathbf{u_D} + (\nabla \mathbf{u_D})^{\mathbf{T}}\right]\right\}/\mathbf{2}}\right|^{n-1}$ [Pa.s]
μ	Dynamic viscosity [Pa.s]
$\overline{\eta} - \xi$	Generalized coordinates
ρ	Specific mass [kg/m^3]
ϕ	Porosity
φ	General dependent variable

Subscripts

CF	Clear Fluid
PM	Porous Medium

Superscripts

i	Intrinsic (fluid) average
v	Volume (fluid + solid) average

and a porous medium. de Lemos and Silva (2006) [18] studied the effect of shear stress jump coefficient on the turbulent fluid flow in channel partly filled with porous material. The results indicated that depending on the value of the stress jump parameter, substantially dissimilar fields for the turbulence energy are obtained. Negative values for the stress jump parameter supplied results closer to experimental data for the turbulent kinetic energy at the interface. Chandesris and Jamet (2009) [19] presented a two-step up-scaling approach that allows to derive the jump conditions that must be imposed at the interface to account for transport phenomena in a fluid/porous domain, where the heat flux and temperature jump conditions are related to surface-excess quantities that depend on the interface location. Valdés-Parada et al. (2009) [20] proposed a methodology for the determination of the stress jump coefficient, using volume averaging in a system analogous to the one used by Beavers and Joseph (1967) [21]. Nield and Kuznetsov (2009) [22] presented the study of the unidirectional flow in a parallel-plate channel consisting of three layers, with a transition layer sandwiched between a porous medium and clear fluid. Within the transition layer, the permeability varies linearly across the channel and matches with the outer layer in a continuous fashion. The authors showed that there is the good agreement between the results using their model and the ones presented in the literature. Kuznetsov and Nield (2010) [23] presented an analytical investigation of forced convection in parallel-plate channel partly occupied by a bidisperse porous medium and partly by a clear fluid, in which was employed the Beavers–Joseph boundary condition at the bidisperse porous medium/clear fluid interface. The works of Silva and de Lemos (2011) [24] and Nimvari et al. (2012) [25] investigated the turbulent flow in channel with a centered porous material. These works showed that the increasing the size of the porous material pushes the flow outwards, increasing the levels of turbulent kinetic energy at the macroscopic interface. Barletta and Nield (2011) [26] and Alves and Barletta (2013) [27] studied the onset of the convective instability throughflow of a power-law fluid saturated in a horizontal porous layer heated. The work of Cekmer et al. (2012) [28] studies the fully developed heat and fluid flow in a parallel plate channel partially filled with porous layer. They showed that the decrease in Darcy number causes increase of pressure drop in the channel. Zheng et al. (2012) [29] investigated the Marangoni convection driven by a power-law temperature gradient. They showed that the temperature and the thermal boundary layer decrease as the Marangoni Number increases for Newtonian fluid and non-Newtonian fluid. Li et al., (2012) [30] analyzed the heat transfer by forced convection in power-law non-Newtonian fluids for a circle duct and concluded that the heat transfer behaviors are strongly depending on the value of the power-law index. Valdés-Parada et al. (2013) [31] proposed a methodology to derive the jump conditions for the velocity and the stress at the interface between fluid and porous medium. It is based on the introduction of macroscopic velocity deviations whose integral over the transition layer supplies a jump