International Journal of Thermal Sciences 100 (2016) 145–154

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

# Inverse analysis of spray cooling on a hot surface with experimental data

Sen Yung Lee [a, *], Tsung Wen Huang [b, 1]

[a] Department of Mechanical Engineering, National Cheng Kung University, Tainan 701, Taiwan, ROC
[b] Metal Processing R&D Department, Metal Industries Research & Development Centre, Kaohsiung 81160, Taiwan, ROC

ARTICLE INFO

*Article history:*
Received 13 November 2014
Received in revised form
2 August 2015
Accepted 21 September 2015
Available online 11 November 2015

*Keywords:*
Spray cooling
Inverse heat conduction
Time-dependent boundary conditions

ABSTRACT

This paper proposes a solution method for the inverse spray cooling problem with relatively long cooling time. The entire time domain is divided into several sub-time intervals. By minimizing the mean square error between the experimental data obtained from inside the body and the estimated data from the derived analytical solution of a spray cooling problem with time-dependent boundary conditions, the temperature function at the spray cooling surface in each sub-time interval can be predicted.

Consequently, the temperature distribution and the heat flux over the entire time and space domains can also be obtained. In addition, the integral transform and tedious numerical operations are not required in the proposed solution method. Mathematical and experimental examples are given to illustrate the simplicity, efficiency, and accuracy of the proposed method.



## 1. Introduction

Inverse heat conduction problems (IHCPs) arise in many heat transfer situations when experimental difficulties are encountered in measuring or producing the appropriate boundary conditions. Practical applications are the estimation of the temperature and the heat flux at the surface of the body under investigation. Rapid cooling and quenching in metal foundries, cooling of electronics components are typical examples of spray cooling.

The present study considers spray cooling on a hot surface. It is well known that under the spray cooling process, the surface temperature of the test hot surface was the most important parameter in quenching and was used to define the distinct heat transfer regimes of the boiling curve. Thus, during the spray cooling process, accurate estimations of the temperature and heat flux on the surface are important. Due to the difficulty of measuring the temperature and heat flux from the spray cooling surface directly, these physical quantities are estimated from the measured temperature data inside the body within the spray cooling time interval. Such estimation is a typical inverse heat conduction problem.

Many numerical techniques have been proposed for solving one-dimensional IHCPs. Among these methods, the finite difference method, the finite element method, and the boundary element method are the numerical tools of choice for the modeling and simulation of IHCPs. Lesnic and Elliott [1] employed Adomian's decomposition and the mollification method to deal with noisy input data and obtained a stable approximate solution. Monde and Mitsutake [2], Monde et al. [3] and Woodfield et al. [4] developed an analytical method using the Laplace transform and half polynomial series of time with a time lag to estimate thermal diffusivity, surface temperature, and heat flux for one-dimensional IHCPs. They recommended choosing the measurement points as close to the surface as possible to obtain a good estimation. Hon and Wei [5], Jin and Zheng [6], and Yan et al. [7] developed meshless and integration-free numerical schemes based on the use of the fundamental solution as a radial basis function for one-dimensional IHCPs. However, the resulting matrix equation is complicated and it is difficult to obtain accurate results. With experimental data, Qiao and Chandra [8] and Cui et al. [9] applied the sequential function specification method to estimate the surface heat flux. Hsieh et al. [10] applied the transient liquid crystal technique and thermocouple to determine the variation of the surface temperature with time during spray cooling of a hot surface for pure water and R-134a. Chen and Lee [11] proposed a hybrid technique of the Laplace transform and the finite difference method in conjunction with experimental temperature data inside a test cylinder to predict the

* Corresponding author. Tel.: +886 6 2757575x62150.
*E-mail addresses:* sylee@mail.ncku.edu.tw (S.Y. Lee), fir@mail.mirdc.org.tw (T.W. Huang).
[1] Tel.: +886 7 3513121x2454.

http://dx.doi.org/10.1016/j.ijthermalsci.2015.09.020
1290-0729/

**Nomenclature**

| | |
|---|---|
| $B_n(\tau)$ | dimensionless quantity defined in equation (31) |
| $B_{nSj}(\tau)$ | dimensionless quantity defined in equation (A9) |
| $c$ | specific heat (W s/kg °C) |
| $C_{jS}$ | coefficients of dimensionless time-dependent function |
| $\mathbb{C}$ | vector in matrix equation |
| $\bar{E}_S$ | function representing the error within each spray cooling sub-time interval |
| $f(t)$ | time-dependent temperature function at the spray cooling surface |
| $\bar{f}(\tau)$ | dimensionless time-dependent function |
| $f_0(t)$ | time-dependent function for first-kind boundary condition |
| $\bar{f}_0(\tau)$ | dimensionless time-dependent function for first-kind boundary condition |
| $f_S(t)$ | time-dependent function at the spray cooling surface within sub-time interval |
| $\bar{f}_S(\tau)$ | dimensionless time-dependent function at the spray cooling surface within sub-time interval |
| $f_{0S}(t)$ | time-dependent function for first-kind boundary condition |
| $\bar{f}_{0S}(\tau)$ | dimensionless time-dependent function for first-kind boundary condition |
| $F(\xi,\tau)$ | dimensionless quantity defined in equation (23) |
| $g_1(\xi)$, $g_2(\xi)$ | shifting functions |
| $k$ | thermal conductivity (W/m °C) |
| $L$ | length of cylinder (m) |
| $p_s$ | number of measured times in $s$-th sub time interval |
| $q(\xi,\tau)$ | dimensionless heat flux |
| $\mathbb{R}$ | vector in matrix equation |
| $N_s$ | number of sub-time intervals |
| $T$ | temperature (°C) |
| $T_r$ | reference temperature (°C) |
| $T_{0S}(x)$ | initial temperature (°C) for $S$th sub-time interval |
| $T_S^{mea}(x_m, t_r)$ | temperature measured at ($x_m$, $t_r$) for $S$-th sub-time interval |
| $t$ | time variable (s) |
| $t_r$ | time of temperature measurement |
| $U_m$ | mean droplet impact velocity, m/s |
| $x$ | spatial-domain variable (m) |
| $\mathbb{Z}$ | matrix in matrix equation |

*Greek symbols*

| | |
|---|---|
| $\alpha_n$ | dimensionless quantity defined in equation (33) |
| $\delta_n$ | norm of $n$-th eigenfunctions |
| $\varepsilon_n$ | dimensionless quantity defined in equation (A6) |
| $\phi_n(\xi)$ | $n$-th eigenfunction |
| $\gamma_n(\tau)$ | dimensionless quantity defined in equation (32) |
| $\eta_{jS}(\xi,\tau)$ | dimensionless quantity defined in equation (A8) |
| $\eta_{jSr}(\xi)$ | dimensionless quantity defined in equation (A8) as $\tau = \tau_r$ |
| $\lambda_n$ | $n$-th eigenvalue |
| $\nu(\xi,\tau)$ | transformed function |
| $\theta$ | dimensionless temperature |
| $\theta_{0S}$ | dimensionless initial temperature for $S$-th sub-time interval |
| $\theta_S^{mea}(\xi_m, \tau_r)$ | dimensionless temperature measured at ($\xi_m$, $\tau_r$) for $S$-th sub-time interval |
| $\rho$ | mass density (kg/m$^3$) |
| $\tau$ | dimensionless time variable |
| $\tau_r$ | dimensionless measured times |
| $\xi$ | dimensionless spatial-domain variable |

spray cooling surface temperature. Lee and Huang [12] introduced a hybrid inverse scheme involving the analytic solution, least-square methods in conjunction with experimental data inside the test material, to study inverse laser surface heating problem. The heat treatment time is short and takes less than 7 s.

Most existing solution methods have to deal with tedious numerical problems, such as the inverse Laplace transform, stability in numerical schemes, and large numbers of cells or elements in matrix operations. In addition, it is recommended that the temperature measuring point be close to the heat treatment surface.

In the present study, we extend the previous study [12] and study the inverse spray cooling problem with relatively long heat treatment time. The entire time domain is divided into several sub-time intervals, depending on the experimental data. Also, by minimizing the mean square errors between the measured experimental data obtained from inside the body and the estimated data from the derived analytical solution form, the unknown temperature function at the spray cooling surface, in polynomial function form, can be determined. Consequently, the temperature distribution and the heat flux within each sub-time interval and space domains can also be obtained. The proposed solution method does not require integral transform and tedious numerical operations. Mathematical and experimental examples are given to illustrate the analysis. The developed solution method is simple, efficient, and accurate. It can be applied to problems with various kinds of time-dependent boundary conditions.

## 2. Mathematical formulation

Consider the spray cooling of the surface of a 25.4 mm diameter copper cylinder, as shown in the schematic diagram in Fig. 1, from Qiao and Chandra's work [8]. To reduce the heat loss, the heater block and the sides of the cylindrical test surface are insulated with mineral wool. The material properties of the cylinder are constants. Four 0.5 mm diameter K-type (chromel–alumel) thermocouples are applied to measure the temperatures of the test cylinder at four different locations of $x_1 = 0.4$ mm, $x_2 = 6.75$ mm, $x_3 = 13.1$ mm and $x_4 = 19.85$ mm. Fig. 1 shows a schematic diagram of the inverse problem. The lower end of the cylinder was bolted to a copper heater block that housed two 500 W cartridge heaters, which were regulated by a temperature controller, holding the surface temperature constant before water was sprayed on it. The mathematical formulation, basic assumptions and experimental temperature data used in this study come from the works of Qiao and Chandra [8] and Cui et al. [9]. The time-dependent temperature function, $f(t)$, at the spray cooling surface is to be determined. A time-dependent temperature boundary condition is applied at the other end $f_0(t)$.

The governing differential equation and the boundary conditions of the system are:

$$k\frac{\partial^2 T(x,t)}{\partial x^2} = \rho c \frac{\partial T(x,t)}{\partial t}, \quad 0 < x < L, \quad t > 0 \tag{1}$$