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Wall effects on the stability of convection in an infinite vertical layer

M. Tadie Fogaing [a], L. Nana [b], O. Crumeyrolle [a, *], I. Mutabazi [a]

[a] LOMC, UMR 6294, CNRS-Université du Havre, B.P. 1123, F-76063 Le Havre Cedex, France
[b] Department of Physics, Faculty of Science, Université de Douala, B.P. 24157, Douala, Cameroon

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ABSTRACT

The stability of buoyancy-driven convection in a vertical infinite fluid layer between two rigid walls with different thermal conductivities and thicknesses is presented. Analytical solutions are derived for parallel base flow, for which linear stability analysis predicts the growth of two-dimensional disturbance. The resulting eigenvalue problem was solved using finite-elements method. Neutral stability curves and associated critical Grashof numbers and wavenumbers are supplied for different characteristic parameters of the flow. It is shown that thermal conductivity and thickness of the walls has a weak influence on hydrodynamic modes but a strong influence on thermal modes which become critical for lower values of Prandtl number than in the case of perfect conducting walls.

1. Introduction

Natural convection occurs in fluid submitted to a temperature gradient in the gravitational field. In the classical Rayleigh–Bénard convection, the temperature gradient is parallel to the gravity, so that there is no base flow and the natural convection is the result of the instability when the Archimedean buoyancy overcomes the viscous and thermal dissipations [1,2]. In case of a fluid confined inside a vertical slot with lateral walls maintained at different temperatures, the temperature gradient is perpendicular to the gravitational field. This generates a shear flow ascending near the hot surface and descending near the cold one. This shear flow has a baroclinic vorticity, orthogonal to the flow plane. For a weak temperature difference between the walls, the profile of the vertical velocity is a cubic polynomial, the temperature distribution is linear and the heat is transferred across the slot by conduction [3]. When the temperature difference exceeds a critical value, the flow becomes unstable and a secondary vortex flow appears. The critical value of the temperature difference and the form of the critical modes depend on the fluid properties, more precisely on its Prandtl number. The presence of secondary vortices in the gap between the walls enhances the heat transfer in the vertical slot. Because of its considerable scientific interests and industrial applications, the problem of the natural fluid convection in a slot with a fixed temperature difference between the vertical side-walls continues to attract much attention of researchers [4–15], with a continuously increasing level of effort devoted to experimental, analytical and computational studies.

The linear stability analysis of convective flow between plane-parallel and vertical conductive impermeable walls has been well documented and the results depend on the Prandtl number (Pr). For $Pr < 12.45$ the critical modes are called hydrodynamic as they result from the destabilization of the velocity profile which possesses an inflection point with a maximum of vorticity. So according to Rayleigh–Fjørtoft criterion [16], the flow is unstable to transverse modes. As there is no mean velocity in the gap between the walls, the critical modes are stationary. For $Pr > 11.56$, a second branch of stability occurs and corresponds to oscillatory thermal modes due to the destabilization of temperature profile [17,18]. These modes become critical modes for $Pr > 12.45$. The intersection point of the two stability branches in the plane spanned by Pr, Gr corresponds to the coexistence of critical hydrodynamic and thermal modes at the point $Pr = 12.45$, $Gr_c = 7872.73$. Such points where two different critical modes coexist, have been found in many systems with two independent control parameters and are referred as to codimension two points [19–22]. In most of studies on natural convection between vertical walls maintained at different temperature, the thickness of both the walls and their thermal conductivities are not often taken into account, assuming that the boundaries are perfect heat conductors. However, in real situations, the flow boundaries have finite thermal conductivities and finite thicknesses.

* Corresponding author.
E-mail address: olivier.crumeyrolle@univ-lehavre.fr (O. Crumeyrolle).

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Nomenclature

d	thickness of the fluid layer
D	derivative operator along x
e_{p_i}	thickness of the solid vertical walls i where $i = 1, 2$
g	gravitational acceleration
Gr	Grashof number
k_f	thermal conductivity of the fluid
k_i	thermal conductivity of the vertical wall i
c_{p_f}	specific heat capacity of the fluid
c_{p_i}	specific heat capacity of the vertical wall i
P	pressure
Pr	Prandtl number
q	dimensionless wavenumber
$r_i = k_f/k_i$	ratio of thermal conductivities
T	temperature
T_0	reference temperature of the fluid
u, w	velocity components in x, z directions
u', w'	components of velocity perturbation
x	horizontal coordinate
z	vertical coordinate

Greek symbols

α	coefficient of thermal expansion of the fluid
ν	kinematic viscosity of the fluid
κ	thermal diffusivity of fluid
ζ_i	ratio of fluid to wall volumetric heat capacities
ρ	density of fluid
ρ_i	density of wall i
σ	real temporal growth rate of the perturbation
ω	frequency of the perturbation
$\delta_i = e_i/d$	ratio of thickness
$\delta\theta$	total temperature difference
δT^*	temperature difference across the fluid layer
$\theta = (T{-}T_0)/\delta T^*$	temperature deviation from the reference
π	generalized pressure term
θ'	temperature perturbation
π'	pressure term disturbance
Δ	Laplacian operator

Subscripts

p	wall
f	fluid
c	critical value
b	base flow
1	left wall
2	right wall

In numerical simulations of flow and heat transfer of forced convection in a micropolar fluid flowing along a vertical slender hollow circular cylinder with wall conduction and buoyancy effects, Chang [23] has showed that the conjugate heat transfer parameter has a significant influence on fluid flow and heat transfer characteristics. Between heated vertical walls, the channel flow, where the velocity profile features a non-zero mean, is also known to be influenced by the thickness of the wall [24,25]. The influence of the boundaries on the onset of Rayleigh–Bénard convection has been investigated by Cérisier et al. [26]. They have shown that the thickness and the thermal conductivities of the wall induce a drastic destabilizing effect on the natural convection flow: the critical values of the Rayleigh number and wavenumber decrease strongly with the increase of the ratio of the thermal conductivity of the fluid to that of the wall. Recently, Mojtabi and Reeds have made a detailed theoretical analysis of the effect of conducting boundaries on the natural convection in the porous layer heated from below bounded by thin horizontal plates of given thickness and with an constant heat flux [27] and with a background flow [28]. For the flow inside a vertical slot, Gershuni et al. [29] report results for the case when the fluid thermal conductivity is much larger than that of walls. In that case, no significant effect was observed on hydrodynamic modes while thermal modes were found at low values of Pr down to $Pr = 0.89$. Recent direct numerical simulations in multi-pane windows by Arıcı et al. [15] did included computational domains for the glass panes, but did not investigated the influence of the glass thickness itself.

The aims of the present note are to report the results of linear stability analysis of the flow inside a vertical slot with differentially heated walls of finite thicknesses and finite thermal conductivities. The critical parameters of the onset of instability are computed for different values of the ratio of the wall thicknesses and thermal conductivities.

The note is organized as follows: in Section 2, we formulate the mathematical description of the flow, determine the base flow profiles and parameter ranges, in Section 3 are derived the linearized perturbation equations around the base flow state, The results are presented in Section 4, and the last section contains discussion and conclusion.

2. Problem formulation

We consider a Newtonian incompressible fluid of density ρ confined in a vertical plane slot of thickness d, bounded by two rigid walls of infinite lateral extension. The two walls have different thicknesses e_1, e_2 and different thermal conductivities k_1, k_2 (the subscripts 1 and 2 stand for the left and the right walls, respectively). The physical model and coordinate system are shown in Fig. 1. The outer surfaces of the left and right walls are maintained at uniform and constant temperatures T_1 and T_2 respectively. We note

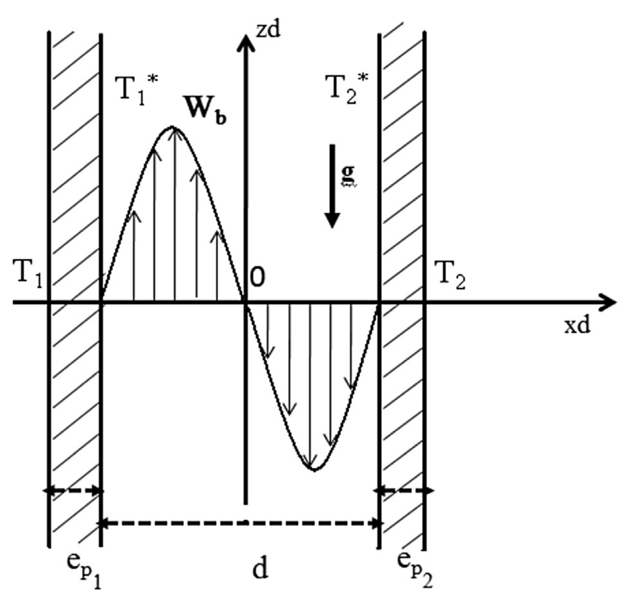

Fig. 1. Schematic representation of the flow configuration. The velocity profile of the base flow is also represented.