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A compact closed-form Nusselt formula for laminar longitudinal flow between *rectangular/square* arrays of parallel cylinders with *unequal* row temperatures

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A R T I C L E I N F O

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A B S T R A C T

Axial flows over cylinders are frequently encountered in practice, e.g. in tubular heat exchangers and reactors. Using the Integral method, closed-form relationships are developed for heat transfer coefficients or Nusselt number inside a fluid flowing axially between a *rectangular/square* array of parallel cylinders with *unequal* temperatures. The model considers the temperature variations of cylinders from one row to another while assuming the same temperature for all the cylinders in each row. The model could well capture several sets of numerical data, which can be regarded as excellent in light of the simplicity and comprehensiveness of the model. The compact and accurate formulae developed in this work can be readily employed, and also implemented into any software or tools, for the estimation of *Nu* in tubular heat exchangers, fins systems, porous media and composite manufacturing.

1. Introduction

Heat transfer through parallel cylinders or tube assemblies is a problem of considerable interest in a variety of industrial thermal applications such as multi-tubular heat exchangers, fins, porous media and rod-bank generators, to name a few [1–3]. A general, easy-to-use and still accurate model that can predict the heat transfer coefficient under different operating conditions is essential for the modeling and design of such systems.

Considerable attempts have been made to study the heat transfer of a longitudinal flow between parallel cylinders. However, almost all of these studies are confined to either numerical solutions or asymptotic models developed for two limited cases: *square* or *triangular* array of cylinders having the *same* temperatures. Table 1 summarizes all the studies performed on laminar-flow heat transfer to a fluid flowing axially between parallel cylinders.

To the authors' knowledge, and as shown in Table 1, the literature lacks a model for estimating the heat transfer coefficient of a laminar flow inside a *rectangular* array of cylinders. Especially, no model or data is available for axial flow of a fluid between parallel cylinders with *unequal* row temperatures, which is indeed a more realistic case in comparison to the case of equal temperatures of the cylinders. The aim of this study is to develop a general compact analytic model for predicting the heat transfer coefficient of a longitudinal fluid flow passing through a rectangular array of parallel cylinders with unequal row temperatures.

It should be noted that the determination of the exact temperature profile is not the final aim of this study. Here we are interested in finding a closed-form analytic relation for the prediction of the heat transfer coefficient. As a result, the integral method can be useful, as it usually leads to compact, simple and sufficiently accurate relations, especially for estimating wall fluxes and average profiles [7–14]. In turn, we use the integral method as a powerful technique for obtaining approximate still reasonable solutions to rather complex problems with remarkable ease. The basic idea is

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Table 1
Review on the works tailored for modeling the heat transfer by laminar axial flow between parallel cylinders: There is no model for the case of unequal cylinders temperatures and/or for *rectangular* arrays of cylinders.

Author(s) & year	Limitations & remarks			
	Array of cylinders	Temperatures of cylinders	Porosity	Type of study
Szaniawski & Lipnicki (2008) [1]	Square	Equal	Very high (>95%)	Analytic (complex series form)
Miyatake & Iwashita (1990) [2]	Square Triangle	Equal	–	Numerical
Antonopoolos (1985) [4]	Rectangle	Equal wall heat flux	–	Numerical
Yang (1979) [5]	Square	Equal	–	Numerical
Sparrow et al. (1961) [6]	Triangle	Equal	–	Analytic, series form

that, from the physics of the system, we assume a general shape of the temperature profile. It must be noted that we are not interested in the precise shape of the temperature profile but rather need to know the heat flux and the average profile of the temperature over the considered domain to calculate the heat transfer coefficient. As mentioned earlier, estimating the average profile and the flux values can be well performed by the integral method. The integral method has been successfully applied to several classical problems such as moving plate and boundary layer [7–14]. However, the use of this method to develop a heat transfer model for the fluid flow between parallel cylinders is a novel approach. In the following sections, the model will be presented in a general form to be also suitable, with only minor changes, to other possible applications such as catalytic reactors and the beds packed with cylindrical materials.

2. Model development

Fig. 1 shows a fluid flowing through parallel cylinders of diameter d and length L extended along the x-direction and spaced in a rectangular array. The spacing between the cylinders centers is H in the vertical (z) direction and is W in the horizontal (y) direction. In this model, contrary to available similar studies as listed in Table 1, W is not necessarily equal to H and these parameters can take different values ($H \geq W$). In other words, the general case of longitudinal flow through a *rectangular* array of parallel cylinders is considered. The model also assumes fully developed steady state, laminar (creeping) [4,6,14–17] incompressible flow. The physical properties are assumed to be constant, and dissipation, gravity and buoyancy effects are negligible. The cylinders temperatures can change from one row to another ($T_{wi} \geq T_{wi+1}$ where i denotes the row number) but are assumed to be the same in each row. The temperature and velocity of the fluid at the inlet is T_∞ and u_∞, respectively ($T_{w1} > T_\infty$).

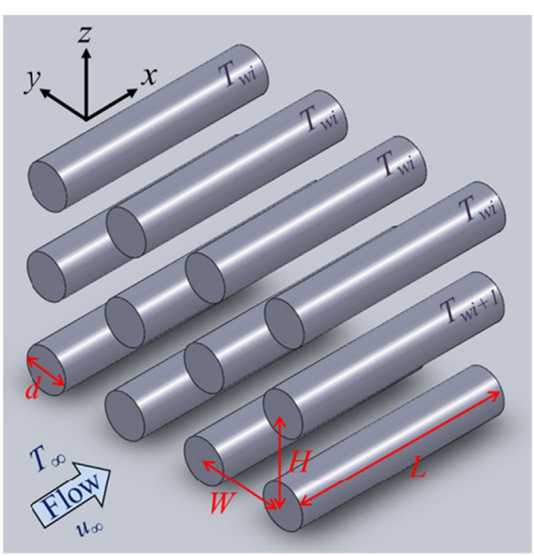

Fig. 1. Fully developed laminar flow between a rectangular array of parallel cylinders; the coordinate system (x,y,z), the fluid temperature and velocity (T_∞ and u_∞), the temperatures of each row of cylinders (T_{wi}: Temperature of all the cylinders in the ith row) and the geometrical parameters W, L, H, and d are shown on figure ($H \geq W$).

Fig. 2 shows the lateral and front views of a longitudinal flow between parallel cylinders and the spacing (δ) between the upper and lower boundaries of the control volume considered:

$$\delta = \begin{cases} \dfrac{H}{2} & 0 \leq y < \dfrac{W-d}{2} \\ \dfrac{H}{2} - \sqrt{\dfrac{d^2}{4} - \left(\dfrac{W}{2} - y\right)^2} & \dfrac{W-d}{2} \leq y \leq \dfrac{W}{2} \end{cases} \tag{1}$$

The analytic expression of the velocity profile obtained by Sparrow and Loeffler [18] shows that the velocity is almost uniform except for the area at the vicinity of the cylinders surfaces. This point is also confirmed by the Fluent simulation results shown later in the "Model verification" section. For this reason, the average velocity ($\bar{u}$), which can be readily obtained from the mass flow rate [2,3], is used in the model derivation throughout this study.

The energy equation and the corresponding boundary conditions are ($T(x = 0,z) = T_\infty$):

$$\bar{u}\frac{\partial T}{\partial x} = \alpha\left(\frac{\partial^2 T}{\partial z^2}\right) \tag{2}$$

$$T\left(x, z = +\bar{\delta}\right) = T_{w1} \tag{3}$$

$$T\left(x, z = -\bar{\delta}\right) = T_{w2} \tag{4}$$

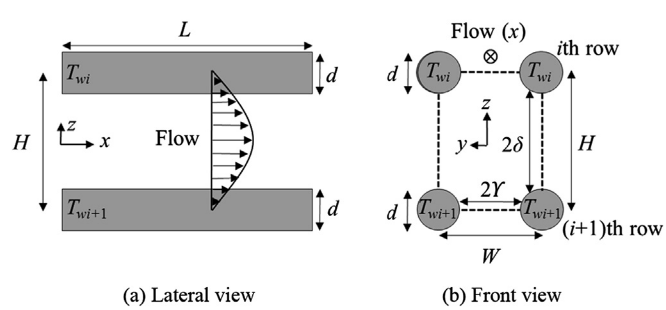

Fig. 2. Front and lateral views of four parallel cylinders with the axial flow; the present model considers the general case of *rectangular* ($H \geq W$) arrays of cylinders which can have unequal row temperatures ($T_{wi} \geq T_{wi+1}$).