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Effects of variable thermal conductivity on Stokes' flow of a thermoelectric fluid with fractional order of heat transfer

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ABSTRACT

In this study, the constitutive relation for the heat flux vector is derived to be the Fourier's law of heat conduction with a variable thermal conductivity and time-fractional order. The Stokes' flow of unsteady incompressible thermoelectric fluid due to a moving plate in the presence of a transverse magnetic field is molded. Stokes' first problem is solved by applying Laplace transform with respect to time variable and evaluating the inverse transform integrals by using a numerical approach. Numerical results for the temperature and the velocity distributions are given and illustrated graphically for given problem. The results indicate that the thermal conductivity and time-fractional order play a major role in the temperature and velocity distributions.

1. Introduction

Heat transfer continues to be a field of major interest to engineering and scientific researchers, as well as designers, developers, and manufacturers. Considerable effort has been devoted to research in traditional applications such as chemical processing, general manufacturing, and energy devices, including general power systems, heat exchangers, and high performance gas turbines [1].

A direct conversion between electricity and heat by using thermoelectric materials has attracted much attention because of their potential applications in Peltier coolers and thermoelectric power generators [2]. The interaction between the thermal and magnetohydrodynamic fields is a mutual one, owing to alterations in the thermal convection and to the Peltier and Thomson effects $\Pi = ST$ [3], where Π is Peltier coefficient, S is thermoelectric power and T is the absolute temperature, (although these are usually small). Thermoelectric devices have many attractive features compared with the conventional fluid-based refrigerators and power generation technologies, such as long life, no moving part, no noise, easy maintenance and high reliability. However, their use has been limited by the relatively low performance of present thermoelectric materials [4]. The performance of thermoelectric devices depends heavily on the material intrinsic property; Z, known as the figure of merit and defined by, $Z = \sigma_o S^2/K$ where σ, K and S are respectively the electrical conductivity, thermal conductivity and thermoelectric power or Seebeck coefficient. Increasing of such parameter Z has a positive effect on the efficiency of thermoelectric device. In order to achieve a high figure of merit, one requires a high thermopower S, a high electrical conductivity σ_o, and a low thermal conductivity K. However, this process is not easy as the written sentence. The direct proportion between σ_o and K, and the inverse proportion between S and σ_o yield a difficulty in improving the efficiency.

Liquid metals are considered to be the most promising coolants for high temperature applications like nuclear fusion reactors because of the inherent high thermal diffusivity, thermal conductivity and hence excellent heat transfer characteristics. Lithium is the lightest of all metals and has the highest specific heat per unit mass. Lithium is characterized by large thermal conductivity and thermal diffusivity, low viscosity, low vapor pressure. Liquid metal in a closed container made of dissimilar metal under a magnetic field is, in general, set into motion by thermoelectric effects if the

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Nomenclature

$\boldsymbol{B}$	magnetic induction vector
$\boldsymbol{q}$	heat flux vector
C_p	specific heat at constant pressure
F_i	Lorentz force
$\boldsymbol{H}$	magnetic field intensity vector
H_o	constant component of magnetic field
$\boldsymbol{J}$	conduction electric density vector
K	Thermal conductivity
K_o	Thermal conductivity at temperature T_0
P_r	Prandtl number
S	Seebeck coefficient
s_o	Seebeck coefficient at temperature T_o
t	time
T	temperature
T_w	temperature of the plate
T_∞	temperature of the fluid away from the plate
U	the standard speed, $[U] = m/sec$
M	$= \frac{\upsilon}{\rho}\frac{\sigma_o B_o^2}{U^2}$, magnetic number
$\boldsymbol{u}$	components of velocity vector (u,v,w)
$\boldsymbol{x}$	space coordinates (x, y, z)

Greek letters

ρ	density
α	fractional derivative parameters
κ	thermal diffusivity
τ_{ij}	$=\mu(u_{i,j}+u_{j,i})$, stress components
μ_o	magnetic permeability
μ	dynamic viscosity
υ	$=\mu/\rho$ kinematic viscosity
σ_o	electrical conductivity
Π	Peltier Coefficient
π_o	Peltier Coefficient at temperature T_0
τ_o	relaxation time

interfacial temperature is nonuniform, a situation likely to occur in fusion reactor blankets owing to the high thermoelectric power of lithium. Lithium is the most promising coolant for thermonuclear power installations. Shercliff [5] treats Hartmann flow and points out the relevance of thermoelectric magnetohydrodynamic (MHD) in liquid metal use, such as lithium, in nuclear reactors.

Accurate measurements of instantaneous local velocities in liquid metals at elevated temperatures are difficult to perform because the most precise techniques are either problematic or impossible to use. Optical methods like laser Doppler anemometry (LDA) are obviously not applicable because sodium is opaque. Hot-film anemometers with quartz-coated probes are not usable at the high temperatures considered in this experiment and their performance is hampered by the high thermal conductivity of liquid metals and impurities deposited on the probe surface, leading to an unpredictable transfer function of the probe. However, some measurements have been reported where this technique was applied at room temperature in mercury [6,7].

Mean and fluctuating velocities in sodium mixing layer experiment have been measured in a fluid with very low Prandtl number ($Pr << 10^{-2}$), with a miniature permanent-magnet velocity probe in the presence of strong temperature gradients by Kapulla et al. [8].

Other techniques which have been used to inspect the velocity field are potential probes in external magnetic fields [7,9], self-contained electromagnetic [10] and permanent-magnet probes [11], pitot tubes [12], fiber-optic sensors [13], pulsed ultrasonic Doppler anemometers (UDA) [14] and temperature fluctuation correlations ('time-of-flight methods') [15].

Investigation of heat transfer, in particular, free-convection heat transfer, involves measuring a heat flux on a surface along which a liquid moves. Two methods are mainly used to measure the heat flux. One of these methods is based on a detailed measurement of the average temperature profile in the immediate vicinity of the surface, whereupon one calculates the temperature gradient and the heat flux. The other method involves the use of diverse heat flux sensors (HFS). The design and features of application of a heat-flux sensor whose operation was based on the transverse Seebeck effect has been described by Mityakov at el [16].

The electrothermal flow is an important phenomenon that occurs in microfluidic systems. The electrothermal flow phenomena can be applied to many microfluidic devices such as lab-on-a-chip. As a result of the small length scale in these devices, the fluid flow is characterized by a low Reynolds number thus allowing the governing equations to become linear. The boundary element method was used to model the 2D electrothermal flow system by Ren at el [17].

Mathematical modeling is the process of constructing mathematical objects whose behaviors or properties correspond in some way to a particular real-world system. The term real-world system could refer to a physical system, a financial system, a social system, an ecological system, or essentially any other system whose behaviors can be observed. In this description, a mathematical object could be a system of equations, a stochastic process, a geometric or algebraic structure, an algorithm or any other mathematical apparatus like a fractional derivative, integral or fractional system of equations. The fractional calculus and the fractional differential equations are served as mathematical objects describing much real-world system.

Nowadays the dynamics of impurities in unsteady flows is quite relevant as shown by several publications, whose aim is to provide more general expressions for the hydrodynamic forces, including the Basset force, in order to fit experimental data and numerical simulations.

Recall the general equation of motion for a spherical particle, in a viscous fluid, pointing out the different force contributions due to effects of inertia, viscous drag and buoyancy. In particular, the so-called Basset force is interpreted in terms of a fractional derivative of order 1/2 of the particle velocity relative to the fluid. The generalized Basset force, which is expressed in terms of a fractional derivative of any order α ranging in the interval $0 < \alpha < 1$ is introduced [18]. This generalization, suggested by a mathematical speculation, is expected to provide a phenomenological insight for the experimental data. The solution for the particle velocity in terms of Mittag-Leffler -type functions. The most evident effect of this generalization is to modify the long-time behavior of the solution, changing its algebraic decay from $t^{-1/2}$ to $t^{-\alpha}$. This effect can be of some interest for a better fit of experimental data.

The classical Langevin equation addresses the dynamics of a Brownian particle through Newton's law by incorporating the effect of the Stokes fluid friction and that of thermal fluctuations in the vicinity of the particle into a random force [19]. Recently, a great interest on the subject matter has been raised because of the