International Journal of Thermal Sciences 49 (2010) 1019–1025

Contents lists available at ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Thermoacoustic energy analysis of transverse-pin and tortuous stacks at large acoustic displacements

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ARTICLE INFO

Article history:
Received 2 August 2009
Received in revised form
7 December 2009
Accepted 12 December 2009
Available online 13 January 2010

Keywords:
Heat–sound interactions
Thermoacoustics
Transverse-pin stack
Tortuous stack
Rayleigh's criterion
Lagrangian framework

ABSTRACT

A simplified method based on Rayleigh's criterion is developed for evaluating thermoacoustic power conversion in transverse-pin and tortuous stacks. Heat transfer and viscous losses are approximated by steady-flow correlations valid at large acoustic displacements with respect to a longitudinal pitch of a pin stack or a characteristic pore size of a random stack. A Lagrangian approach is employed to calculate temperature fluctuations of oscillating gas parcels inside the stack. A computational example is presented for a stack with an inline pin arrangement placed in a standing acoustic wave. Power conversion and efficiencies are evaluated for conditions relevant to a small-scale system. An indirect comparison is also made between theoretical results and experimental data for a prime mover with a wire mesh stack.

1. Introduction

Thermoacoustic engines generate sound from heat without moving solid parts. Acoustic modes inside the engine resonators become unstable if a sufficiently high-temperature gradient is imposed in a porous insert, known as stack, which should be properly positioned inside the system. A schematic of a simple standing-wave thermoacoustic prime mover is shown in Fig. 1. In-stack gas parcels undergoing motions in a fundamental acoustic mode receive heat from the stack in the moments of their compression and reject heat back to the stack in the rarefied states. Such a heat exchange transforms part of supplied heat into sound in accordance with Rayleigh's criterion [1]. Acoustic power produced in thermoacoustic engines can be used for a variety of applications [2], such as electricity generation and gas-mixture separation. A thermoacoustic device can also perform as a heat pump or refrigerator, if the temperature difference in the stack is sufficiently small and acoustic oscillations are imposed externally.

Low-amplitude thermoacoustic processes are well understood in stacks with longitudinal parallel-type geometries, where solid components of the stack are aligned in the direction of primary acoustic motions of gas particles [2]. Perhaps the most efficient longitudinal parallel-type configuration involves an array of pins [3], where convex solid surfaces increase thermoacoustic energy conversion while reducing viscous losses.

Stacks with non-uniform cross-sectional geometry comprise wire mesh screens [4], rigid foams [5,6], intermittent plate stacks [7], wools [8], and transverse-pin arrays [9]. Some of these stacks demonstrated a potential of higher performance in comparison with regular stacks that have longitudinal orientation of solid constituents. Another important advantage of non-uniform stacks is significantly reduced heat conduction leak through the stack solid matrix. However, thermoacoustic theories for such geometries are not well developed. Several important studies on thermoacoustics of non-uniform media include a harmonic analysis of stacked screen regenerators [10], measurements and correlations of heat transfer from wires near a thermoacoustic stack [11], and a capillary-tube modeling approach for random porous media [12].

The main objective of the present study is to develop a simplified analysis for thermoacoustic power conversion and the efficiency of this conversion in highly porous, relatively short stacks consisting of an array of pins oriented *perpendicular* to the main acoustic motion of gas. In analogy with the longitudinal pin arrangement, the convex surface of pins is expected to provide additional benefits in this configuration. An important limitation of the theory is that acoustic displacements of gas parcels must be much larger than a longitudinal distance between neighboring pins. In this regime, steady-flow correlations can be applied for

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1290-0729/$ – see front matter
doi:10.1016/j.ijthermalsci.2009.12.007

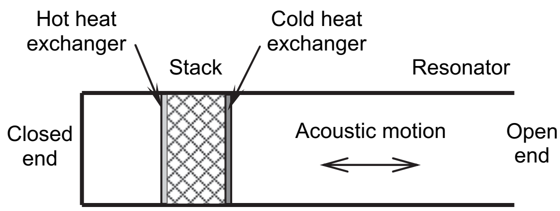

Fig. 1. Schematic of a standing-wave engine.

approximately estimating instantaneous heat transfer and viscous losses [2].

The theory presented here can be also used for high-displacement flow regimes in tortuous stacks if their steady-flow heat transfer and friction correlations are known. Such stacks are employed in miniature thermoacoustic devices [8,13]. A construction of regular-geometry stacks is challenging at small scales. Therefore, the availability of random porous stacks, such as made of reticulated vitreous carbon (RVC), is an important practical advantage, in addition to reduced heat conduction leak and good thermoacoustic performance.

2. Mathematical model

A porous stack with a length L_{st} much smaller than an acoustic wavelength λ is considered. An example of a transverse-pin stack arrangement is shown in Fig. 2. A constant gradient of the mean temperature dT_m/dx is imposed along the stack. The temperature difference between the stack ends, $L_{st}|dT_m/dx|$, is significantly smaller than the mean stack temperature $T_{st,m}$, which is used for evaluating gas properties. The gas inside the stack is represented by a row of moving gas slices (parcels) with fixed mass. Each parcel has a cross-sectional area of the stack and a thickness of one longitudinal pitch of the stack S_L (or characteristic pore size in a tortuous medium) evaluated at the parcel's mean temperature and pressure. The Lagrangian consideration employed here is similar to previous

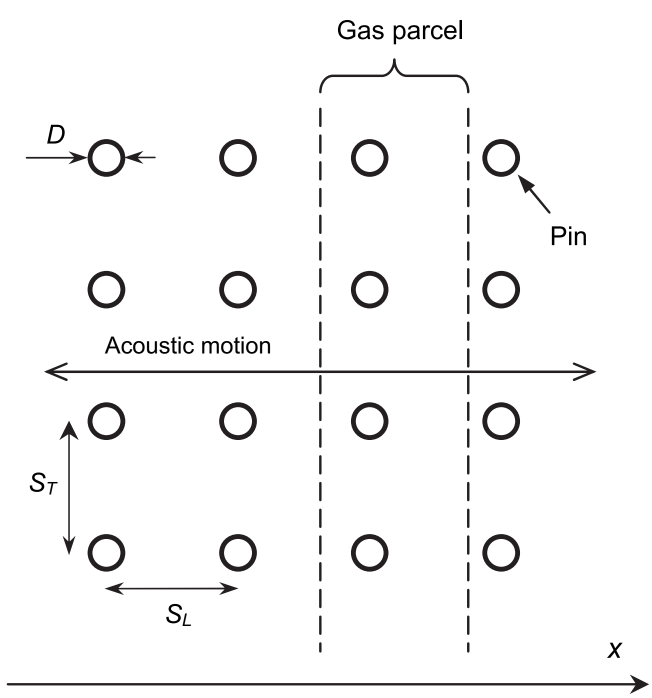

Fig. 2. Schematic of a small part of transverse-pin stack.

thermoacoustic analyses [14,15]. One-dimensional acoustic oscillations along the x-axis are present in the gas. The same acoustic pressure, velocity and displacement will approximate oscillations of all gas parcels inside a high-porosity short stack. Distortions of these parcels are neglected. The acoustic displacement x_1 should be much greater than S_L in order to use steady-flow heat transfer and friction correlations. At the same time, x_1 should be significantly smaller than the stack length L_{st} in order to minimize the influence of the stack ends.

Dynamics of one gas parcel is analyzed. Its acoustic velocity u', displacement x', and pressure fluctuation p' are described by the following equations,

$$u' = u_1 \sin\varphi, \tag{1}$$

$$x' = -x_1 \cos\varphi, \tag{2}$$

$$p' = p_1 \sin(\varphi + \theta), \tag{3}$$

where φ is the phase variable and θ is the phase shift between pressure and velocity fluctuations. The pressure amplitude is assumed to be relatively small in comparison with the mean pressure. It is also assumed that acoustic oscillations are monochromic, and the frequency of oscillations is determined by the resonator characteristics. The relation between acoustic pressure and velocity amplitudes depends on the stack position in the acoustic wave,

$$\frac{p_1}{u_1} = \rho_m a_m \overline{\overline{Z}}, \tag{4}$$

where ρ_m and a_m are mean values of the gas density and the speed of sound, and $\overline{\overline{Z}}$ is the absolute value of the non-dimensional specific acoustic impedance.

The evolution of the parcel temperature fluctuation in time (spatially averaged over the parcel) can be modeled as follows [14,15],

$$\frac{dT'}{d\varphi} = \frac{\gamma - 1}{\gamma} \frac{T_m}{p_m} \frac{dp'}{d\varphi} + \frac{1}{mc_p} \frac{dQ}{d\varphi}, \tag{5}$$

where γ is the specific heat ratio, T_m and p_m are the mean temperature and pressure of the gas parcel, m is the parcel mass, c_p is the specific heat at constant pressure, and Q is the heat addition to the parcel. The dominant causes for the gas-parcel temperature oscillations are pressure fluctuations and heat exchange with the stack. Contributions of heat conduction between gas parcels, heat generation due to viscous dissipation, and natural convection to the periodic temperature fluctuations are neglected.

Under the previously stated assumption $x_1 >> S_L$, it can be assumed that the heat transfer coefficient at each time moment depends on the instantaneous flow velocity in form of empirical steady-flow correlations [2,10]. Then, the following quasi-steady function for the convective heat transfer rate can be used,

$$\frac{dQ}{d\varphi} = -\frac{1}{\omega} h(|\mathrm{Re}(\varphi)|) A (T_g - T_s), \tag{6}$$

where ω is the angular acoustic frequency, h is the given steady heat transfer coefficient that depends on the instantaneous Reynolds number, system geometry, and gas properties; A is the reference area; T_g is the gas-parcel temperature; and T_s is the solid surface temperature. The heat transfer coefficient in Eq. (6) depends on the velocity magnitude but not on the frequency. This is in contrast to stacks with ideal longitudinal-pore geometries, where h depends on the frequency but not on the velocity magnitude [16]. In the longitudinal-pore configurations the gas region affected by heat