International Journal of Thermal Sciences 52 (2012) 22–30

Contents lists available at SciVerse ScienceDirect

International Journal of Thermal Sciences

journal homepage: www.elsevier.com/locate/ijts

Inverse estimation of front surface temperature of a plate with laser heating and convection–radiation cooling

Jianhua Zhou, Yuwen Zhang*, J.K. Chen, Z.C. Feng

Department of Mechanical and Aerospace Engineering, University of Missouri, Columbia, MO 65211, USA

A R T I C L E I N F O

Article history:
Received 15 June 2011
Received in revised form
17 September 2011
Accepted 19 September 2011
Available online 19 October 2011

Keywords:
Inverse heat conduction
Radiation and convection boundary condition
Three-dimensional
Conjugate gradient method

A B S T R A C T

The conjugate gradient method (CGM) is an efficient iterative regularization technique for solution of the inverse heat conduction problem (IHCP). However, most of the existing CGM schemes deal with linear boundary conditions and constant thermophysical properties. Little attention has been paid to formulate the CGM with radiation boundary condition and temperature-dependent thermophysical properties. In this study, a nonlinear CGM scheme is formulated to recover the front surface heating condition of a 3-D object, based on the temperature measurements at back surface. The 3-D object is subjected to a high-intensity Gaussian laser beam heating on the front surface and a combined radiation and convection boundary condition on the back surface. The derivations of the direct problem, adjoint problem and sensitivity problem are presented in detail. The results are presented for two materials and excellent agreement between the inverse and exact solutions are demonstrated.

1. Introduction

The inverse heat conduction problems (IHCPs) are mathematically classified as *ill-posed*, so special solution techniques are usually required to transform the ill-posed IHCP into a well-posed one. Although some analytical solutions are available for this purpose (e.g., [1–5]), their application is limited to the IHCPs in one-dimension or simple configuration in 2 or 3 dimensions. For this reason, a number of numerical approaches have been developed for the solution of IHCPs. The interested readers can refer to the books of Tikhonov et al. [6], Beck et al. [7], Alifanov [8] and Özisik [9] for details about these solution techniques.

The majority of the numerical methods restate the inverse problem as a least-squares minimization problem over the whole-time domain or in sequential time intervals. Among those, the conjugate gradient method (CGM) has been receiving more and more attentions since it can improve the convergence rate of inverse estimation by choosing the direction of descent as the linear combination of the gradient direction at current iteration with the direction of descent at previous iteration [10]. Due to its excellent self-adjusting, global convergence property, the CGM has been extensively used to solve multidimensional IHCPs (e.g. [11–14]). Efforts have also been made to develop nonlinear inverse approaches in estimating thermophysical properties and convective heat transfer coefficients at boundaries [15–18]. But most of the algorithms dealt with 1-D or 2-D problems. Recently, Liu proposed to use lie-group shooting and time-marching methods to solve nonlinear inverse problems [19,20]. More work is needed in extending their applications into multidimensional analysis.

Recently, the authors employed the CGM to reconstruct the front-surface heating condition with back-surface heat flux and temperature measurement data with application background in high-energy laser interaction with target [21,22], but only linear boundary conditions are considered. To the best of the author's knowledge, little work has been done to formulate the IHCP with radiation boundary condition and temperature-dependent thermophysical properties in a 3-D configuration. In this study, a 3-D CGM is formulated to reconstruct the front surface heating condition based on the temperature measurement data at the back surface to which a combined radiation and convection boundary condition is imposed. The simulated measurement data are obtained by solving a direct problem in which the front surface is subjected to laser heating as well as convection-radiation cooling and the back surface is subjected to convection–radiation cooling. The robustness of the formulated 3-D IHCP algorithm is tested for two materials that are commonly used in aerospace engineering.

* Corresponding author.
E-mail address: zhangyu@missouri.edu (Y. Zhang).

1290-0729/$ – see front matter © 2011 Elsevier Masson SAS. All rights reserved.
doi:10.1016/j.ijthermalsci.2011.09.009

Nomenclature

C volume specific heat, J/(m^3·K)
$d^k\,(y, z, t)$ direction of descent at iteration k, which is sometimes expressed in vector form $\mathbf{d^k}$
h convection heat transfer coefficient, W/(m^2·K)
i_m total number of temperature measurements
k thermal conductivity, W/(m·K)
L object length in x direction, m
M object length in y direction, m
N object length in z direction, m
q heat flux, W/m^2
q_{laser} periodic laser heat flux on front surface, W/m^2
$q_{\max}$ maximum heat flux at the laser Gaussian beam center, W/m^2
$q_1\,(y, z, t)$ observed heat flux on front surface which is sometimes expressed in vector form $\mathbf{q_1}$, W/m^2
$\Delta q_1(y,z,t)$ heat flux perturbation on front surface
r radius measured from laser spot center, m
S objective function
$\nabla S[q_1^k]$ gradient direction of objective functional at iteration k
$\Delta S[q_1^k]$ objective function variation
t time, s
t_f final time, s
Δt time step, s
T temperature, K
T_0 initial temperature, K
T_∞ ambient temperature, K
$T_1\,(y, z, t)$ front surface temperature, K
$\Delta T[L,y,z,t;d^k]$ temperature variation, which is sometimes simplified as ΔT, when the surface heat flux is perturbation is $\Delta q_1(y,z,t) = d^k(y,z,t)$
w 1/e radius of Gaussian laser beam, m
x, y, z spatial coordinate variables, m
$Y_{TLexact}\,(y,z,t)$ temperature measurement data without errors on back surface obtained by numerical simulations, K
$Y_{TL}\,(y, z, t)$ measurement temperature on the back surface, K

Greek symbols
α surface absorptivity
β^k search step size at iteration level k
χ tolerance used to stop the CGM iteration procedure
δ Dirac delta function
ε surface emissivity
φ standard deviation of temperature measurements, K
γ^k conjugate coefficient at iteration level k
$\lambda\,(x, y, z, t)$ Lagrange multiplier
σ Stefan–Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ W/(m^2·K^4)
ω a random variable having a normal distribution with zero mean and unitary standard deviation
ξ perturbed variable

Superscripts
k iteration level

Subscripts
0 initial
f final
q heat flux
T temperature

2. Model description

A three-dimensional object is considered as shown in Fig. 1. Initially, the object is under a uniform temperature T_0 and then is subjected to a high-intensity Gaussian laser beam q_{laser} (w is the 1/e radius) on the front surface from $t = 0^+$. The purpose of this study is to demonstrate the effectiveness and accuracy of the proposed IHCP formulation in reconstructing the observed heat flux q_1 (y, z, t) and temperature T_1 (y, z, t) on the front surface of a 3-D target with temperature-dependent thermo-physical properties, based on the measured temperature on the back surface.

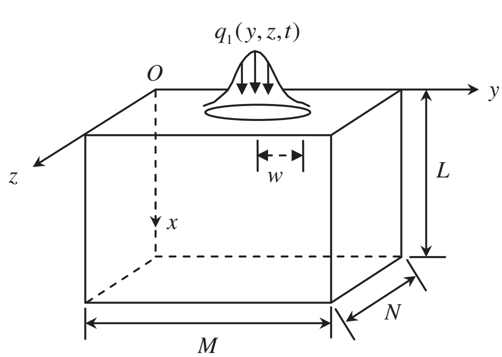

Fig. 1. Physical model.

2.1. Direct problem

The direct problem can be expressed as follows:

$$C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[k(T)\frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[k(T)\frac{\partial T}{\partial z}\right] \quad \text{for } 0 < x < L,\ 0 < y < M,\ 0 < z < N,\ t>0 \tag{1}$$

$$T = T_0 \quad \text{for} \quad 0 \le x \le L,\ 0 \le y \le M,\ 0 \le z \le N,\ t = 0 \tag{2}$$

$$-k(T)\frac{\partial T}{\partial x} = q_1(y,z,t) \quad \text{for} \quad x = 0,\ t>0 \tag{3}$$

$$-k(T)\frac{\partial T}{\partial x} = h(T - T_\infty) + \varepsilon\sigma\left(T^4 - T_\infty^4\right) \quad \text{for} \quad x = L,\ t>0 \tag{4}$$

$$-k(T)\frac{\partial T}{\partial y} = 0 \quad \text{for} \quad y = 0, M;\ t>0 \tag{5}$$

$$-k(T)\frac{\partial T}{\partial z} = 0 \quad \text{for} \quad z = 0, N;\ t>0 \tag{6}$$

where h, ε and T_∞ are assumed to be constant.

In the direct problem described above, the front-surface heat flux $q_1\,(y, z, t)$ is considered to be known. The objective of the direct problem here is to determine the transient temperature and heat flux distribution in the target. As can be seen in Eq. (4), the back surface ($x = L$) is subjected to a radiation boundary condition, which makes the heat conduction a nonlinear problem.