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Instability of Taylor–Couette flow between concentric rotating cylinders

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Abstract

The energy gradient theory is used to study the instability of Taylor–Couette flow between concentric rotating cylinders. This theory has been proposed in our previous works. In our previous studies, the energy gradient theory was demonstrated to be applicable for wall-bounded parallel flows. It was found that the critical value of the energy gradient parameter $K_{\max}$ at turbulent transition is about 370–389 for wall-bounded parallel flows (which include plane Poiseuille flow, pipe Poiseuille flow and plane Couette flow) below which no turbulence occurs. In this paper, the detailed derivation for the calculation of the energy gradient parameter in the flow between concentric rotating cylinders is provided. The calculated results for the critical condition of primary instability (with semi-empirical treatment) are found to be in very good agreement with the experiments in the literature. A possible mechanism of spiral turbulence generation observed for counter-rotation of two cylinders can also be explained using the energy gradient theory. The energy gradient theory can serve to relate the condition of transition in Taylor–Couette flow to that in plane Couette flow. The latter reasonably becomes the limiting case of the former when the radii of cylinders tend to infinity. It is our contention that the energy gradient theory is possibly fairly universal for analysis of flow instability and turbulent transition, and is found valid for both pressure and shear driven flows in parallel and rotating flow configurations.

Keywords: Instability; Transition; Taylor–Couette flow; Rotating cylinders; Energy gradient; Energy loss; Critical condition

1. Introduction

Taylor–Couette flow refers to the problem of flow between two concentric rotating cylinders as shown in Fig. 1 [1–4]. This terminology was named after the works of G.I. Taylor (1923) and M. Couette (1890). This problem was first investigated experimentally by Couette (1890) and Mallock (1896). Couette observed that the torque needed to rotate the outer cylinder increased linearly with the rotation speed until a critical rotation speed, after which the torque increased more rapidly. This change was due to a transition from stable to unstable flow at the critical rotation speed. Taylor was the first to successfully apply linear stability theory to a specific problem, and succeeded in obtaining an excellent agreement of theory with experiments for the flow instability between two concentric rotating cylinders [5]. Taylor's groundbreaking research for this problem has been considered as a classical example of flow instability study [6–8].

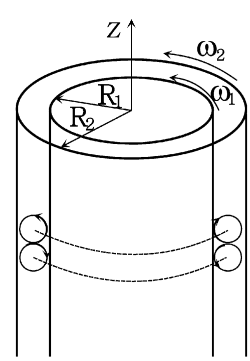

Fig. 1. Taylor–Couette flow between concentric rotating cylinders.

In the past years, the problem of Taylor–Couette flow has received renewed interests because of its importance in flow stability and the fact that it is particularly amenable to rigorous mathematical treatment/analysis due to infinitesimal dis-

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Nomenclature

A, A_a, A^*	coefficients	s^{-1}
$\bar{A}$	amplitude of the disturbance distance	m
B, B_a, B^*	coefficients	$m^2\,s^{-1}$
D	diameter of the pipe for pipe flow	m
E	total mechanical energy of unit volume of fluid	$J\,m^{-3}$
h	$= R_2 - R_1$, gap width between the inner cylinder and the outer cylinder	m
H	total mechanical energy loss of unit volume of fluid due to viscosity in streamwise direction	$J\,m^{-3}$
K	function of coordinates (dimensionless)	
K_c	critical value of $K_{\max}$ for instability (dimensionless)	
$K_{\max}$	maximum of K in the domain (dimensionless)	
l	half-width of the channel for plane Poiseuille flow and plane Couette flow	m
n	coordinate in transverse direction	m
p	static pressure	$N\,m^{-2}$
r	radius	m
R_0	average radius of inner cylinder and outer cylinder	m
R_1	radius of inner cylinder	m
R_2	radius of outer cylinder	m
Re	Reynolds number (dimensionless)	
s	coordinate in streamwise direction	m
t	time	s
T	Taylor number (dimensionless)	
u	velocity component in the main flow direction	$m\,s^{-1}$
u_0	velocity at the mid-plane for plane Poiseuille flow (channel flow)	$m\,s^{-1}$
U	average velocity in the flow passage	$m\,s^{-1}$
v	velocity component in the transverse direction	$m\,s^{-1}$
v'_m	$= \bar{A}\omega_d$, amplitude of the disturbance of velocity in transverse direction	$m\,s^{-1}$
W	work done to the unit volumetric fluid by external	$J\,m^{-3}$
x	coordinate in the streamwise direction	m
y	coordinate in the transverse direction	m
z	coordinate in the spanwise direction	m
η	radius ratio, $\equiv R_2/R_1$	
θ	angular coordinates	rad
λ	speed ratio, $\equiv \omega_2/\omega_1$	
μ	dynamics viscosity	$N\,m^{-2}\,s$
ν	kinematic viscosity	$m^2\,s^{-1}$
ρ	density of fluid	$kg\,m^{-3}$
τ	shear stress	$N\,m^{-2}$
ω	angular velocity of the fluid	$rad\,s^{-1}$
ω_1	angular velocity of the inner cylinder	$rad\,s^{-1}$
ω_2	angular velocity of the outer cylinder	$rad\,s^{-1}$
ω_{1a}	angular velocity of the inner cylinder after splitting	$rad\,s^{-1}$
ω_{2a}	angular velocity of the outer cylinder after splitting	$rad\,s^{-1}$
ω_d	frequency of the disturbance	s^{-1}

turbances [1–3]. For the stability of an inviscid fluid moving in concentric layers, Lord Rayleigh [9] used the circulation variation versus the radius to explain the instability while von Karman [10] employed the relative roles of centrifugal force and pressure gradient to interpret the instability initiation. Their goal was to determine the condition for which a perturbation resulting from an adverse gradient of angular momentum can be unstable. In his classic paper, Taylor [5] presented a mathematical stability analysis for viscous flow and compared the results to laboratory observations. Taylor observed that, for small ratio of the gap width to the cylinder radii and for a given rotating speed of outer cylinder, when the rotation speed of the inner cylinder is low, the flow remains laminar; when the rotation speed of the inner cylinder exceeds a critical value, instability sets in and rows of cellular vortices are developed. When the rotating speed is increased to an even higher value, the cell rows break down and a turbulence pattern is produced. He proposed a parameter, now commonly known as the Taylor number, $T = Re^2(h/R_0)$, to characterize this critical condition for instability. Here, Re is the Reynolds number based on the gap width (h) and the rotation speed of the inner cylinder, and R_0 is the mean radius of the inner cylinder and the outer cylinder. The critical value of the Taylor number for primary instability is 1708 as obtained from linear analysis. This value agrees well with his experiments [1–3]. For Taylor–Couette flow, Snyder has given a semi-empirical equation for the critical condition from the collected experimental data [11]. Esser and Grossmann have also given an analytical equation for the critical condition by an simple approximation, but a constant in the equation have to be fixed using the result of linear stability analysis [12].

However, the problem of Taylor–Couette flow is still far from completely resolved despite extensive study [11–17]. For example, the limiting case of Taylor–Couette flow when the ratio of the gap width to the radii tends to zero should agree with that of plane Couette flow. This includes two possibilities: either radius is infinite or gap width is very small. Thus, the criterion for instability should reflect this phenomenon. There are some recent works trying to address this issue to some degree of success [18–20]. One may observes that Taylor's criterion is not appropriate when this limiting case is studied because plane Couette flow is judged to be always stable due to Taylor number assuming a null value using Taylor's criterion. This may be attributed to the fact that Taylor's criterion only considered the effect of centrifugal force, and does not include the kinematic inertia force. Therefore, it is reckoned to be suitable for low Re number flows with high curvature. For rotating flow with