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Laminar MHD flow and heat transfer of power-law fluids in square microchannels

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ABSTRACT

The steady fully developed laminar flow and heat transfer of electrically conducting non-Newtonian fluids through square microchannels at the presence of transverse magnetic field are investigated with emphasis on viscous dissipation and joule heating effects. The governing continuity, momentum and energy equations are being discretized using a finite difference approach using Modified Power-Law (MPL) model and have been solved iteratively, through Alternating Direction Implicit (ADI) scheme with successive over-relaxation under the classical boundary conditions of no-slip velocity and uniform constant heat flux for flow and thermal fields.

The velocity, temperature profiles, product of friction factor–Reynolds number and Nusselt number are computed for various values of flow index and dimensionless shear rate parameter of modified power-law fluids. Also, the effect of Hartmann number and Brinkman number on velocity, temperature distributions and Nusselt number are discussed. It is shown that the results are in good agreement with the previous studies.

1. Introduction

In recent years, research on characteristics of flow and heat transfer in micro-flow devices (MFDs) has get great attentions due to the rapid development of micro-electromechanical systems (MEMS) and micro-total analysis systems. Microfluidic transport finds its importance in a wide variety of emerging technologies, ranging from the cooling of electronic devices to biomedical diagnostics and biochemical process engineering. Microfluidic usually processes biological and chemical fluids which may be treated as non-Newtonian fluids [1–6]. To obtain the desired functions through microfluidic systems, it is inherently necessary to control the fluid flow and heat transfer in microchannels. More recently Magnetism and microfluidics have been combined in an amazing variety of ways by using the magnetic fields which are usually applied from outside the channel. Magnetohydrodynamic (MHD) is the mutual interaction between the magnetic fields and electrically conducting fluids flow which is used for pumping, controlling and mixing of fluids, as well as the incorporation of switches and valves in to lab-on-a-chip devices [7]. According to aims of this study, the review of literature is presented in three sections:

i. Fluids flow in microchannels and viscous dissipation importance

A large number of experimental and numerical studies that focus on flow and heat transfer behavior in microchannels have been reported. However, most of them are limited to Newtonian fluids. Shah and London [8] presented a rich biography on previous publications in formation of laminar flows in Newtonian fluids. Also, a complete review of experimental and numerical studies on pressure drop and the convective heat transfer in microchannels is offered by Morini [9]. Salman et al. [10] presented the comprehensive review of previous efforts for different convective flow regimes and heat transfer through microtube and microchannel.

Herwig and Hausner [11] suggested that the same equations can be used in order to study the forced convection of liquid in the laminar regime for both macro and micro-flows; nevertheless, certain effects may have different importance for micro-systems in comparison with macro-systems.

Morini and Spiga [12] analytically determined the steady temperature distribution and the Nusselt numbers for a Newtonian incompressible fluid through a rectangular microchannel in fully developed laminar flow with viscous dissipation for any combination of heated and adiabatic sides of the duct in uniform heat flux

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Nomenclature

A_c	cross-sectional area of duct, m^2
A_c^+	dimensionless cross-sectional area of duct
b	duct width, m
b^+	dimensionless duct width
B	magnetic field strength, Tesla
Br	Brinkman number
c	duct height, m
c^+	dimensionless duct height
C_p	fluid specific heat, J/kg K
D_h	hydraulic diameter of duct, m
f	Darcy friction factor
h	convective heat transfer coefficient, W/m^2 K
Ha	Hartmann number
k	thermal conductivity, W/m K
m	consistency index, N s^n/m^2
n	flow index
Nu	Nusselt number
p	perimeter of the channel, m
p^+	dimensionless heated perimeter
P	pressure of the fluid in the duct, Pa
Pr	Prandtl number
q_w	linear heat flux, W/m
Re_{D_h}	Reynolds number
Re_M	modified Reynolds number
T	temperature
T_m	bulk mean temperature
T^+	dimensionless temperature
T_m^+	the bulk mean temperature
u	velocity in flow direction, m/s
u_m	average velocity in flow direction, m/s
u^+, u^{++}	dimensionless velocity in flow direction
x, y, z	rectangular Cartesian coordinates
x^+, y^+, z^+	dimensionless coordinates in x, y, z axes

Greek symbols

α	aspect ratio of rectangular duct
β	shear rate parameter
η	reference viscosity, N m/s^2
η_0	zero shear rate viscosity, N m/s^2
η_a	apparent viscosity, N m/s^2
$\eta_{a,x}$	apparent viscosity, N m/s^2
$\eta_{a,x}^+$	dimensionless viscosity
$\eta_{a,y}$	apparent viscosity, N m/s^2
$\eta_{a,y}^+$	dimensionless viscosity
ρ	fluid density, kg/m^3
Φ^+	dimensionless viscous energy dissipation defined by Eq. (26)

Subscripts

$+$	refers to dimensionless quantities
$++$	refers to dimensionless quantities

boundary condition. They demonstrate that the problem of heat transfer enhancement in micro-devices cannot be solved by indefinitely reducing the microchannel dimensions because the viscous dissipation effects shall offset the gains of high heat transfer coefficients associated with a reduction in the channel size. Morini [13] studied the role of viscous heating in fluid flowing through microchannels and developed a model based on convectional theory to predict the viscous dissipation effects.

Gokturk and Bayazitoglu [14,15] studied the convective heat transfer for steady state laminar hydrodynamically developed flow in circular microtubes and rectangular microchannels. They concluded that the Brinkman number plays an important role in the heat transfer through micro-devices.

The effect of viscous dissipation on temperature field and friction factor in microtube and microchannel was investigated by Koo and Kleinstreuer [16]. Their results indicate that the viscous dissipation effect on the friction factor was increased as the system size decreases. They concluded that ignoring viscous dissipation could affect accurate flow simulation in microchannel.

ii. Non-Newtonian fluids flow in microchannels and modeling

The flow and heat transfer behavior of non-Newtonian fluids in microchannels have received considerable attentions due to many important and broad applications in microfluidic systems [17].

Kozicki et al. [18] have proposed two general equations for prediction of flow rate and maximum velocity versus pressure drop relationships in isothermal, steady, uniform, laminar flow of incompressible, time-independent non-Newtonian fluids in ducts of arbitrary cross section. They have presented a new generalized relation between friction factor–Reynolds number that is applicable to all shapes of flow section as well as to all time-independent non-Newtonian fluids.

Wheeler and Wissler [19] studied the friction factor–Reynolds number relation for steady fully developed laminar flow of pseudoplastic fluids through rectangular ducts and applied an overrelaxation method to obtain more accurate velocity profiles and friction factors.

Park and Lee [20] presented numerical solution for fully developed laminar flow for pseudoplastic fluids in conduits of arbitrary cross-sections. In their studies, numerical results of the friction factor-Reynolds number relations compared to the results of previously correlate published works showed that serious errors can result if the incorrect shear rate solution is used.

The power-law model frequently is used in calculation of non-Newtonian fluids flow. However, this model cannot correctly predict the velocity distribution in low shear rates region. To resolve this problem and predict more accurately the velocity field even in the region of lower shear rates including zero shear rate the modified power-law model is proposed by Irvine and Karni [21].

Davaa et al. [22] have studied the effects of viscous dissipation on fully developed heat transfer of non-Newtonian fluids flowing between parallel plates with one of the plates having axial movement. They used the modified power-law model in order to determine the fully developed velocity distribution. Also, a theoretical analysis of the Joule heating effect on a purely electroosmotic flow of non-Newtonian fluids through slit microchannels is presented by Sanchez et al. [23].

iii. Magnetohydrodynamic and applications in microfluidics

The MHD model is a combination of the Navier–Stokes PDE and the Magnetic Induction PDE, which is derived from the Maxwell equations. When an electrically conducting fluid moves in the presence of a transverse magnetic field, it produces an electric field due to charge separation and subsequently an electric current. The interaction between this created electric current and the imposed magnetic field produces a body force, called the Lorentz force which acts on the fluid itself in the opposite direction of the fluid motion. Also, Lorentz Force is generated by applying an electric